Slope stability analysis – limit equilibrium or the finite element method?

Carol Matthews and Zeena Farook, *Arup; and*
Peter Helm, *Newcastle University*

1. Introduction

Since the 1930s, the limit equilibrium (LE) approach has been used to analyse slopes. This approach makes use of a number of differing analysis methods depending on the type of problem (circular vs non-circular) to be solved and the required accuracy of the result.

The initial method adopted for undertaking LE analysis was the Fellenius or Swedish circle method (Fellenius, 1936). This method can only be applied to circular slip surfaces and leads to significant underestimation of the factor of safety (FoS) and is now rarely used. Bishop (1955) developed a revised method for undertaking circular slip analysis which improved the accuracy of the resultant FoS. This revised method required an iterative procedure to solve and so it was suited to computer methods where this could be automated. Bishop's methods are still routinely used in slope stability analysis software to this day.

To undertake analysis of non-circular slips, Janbu's method is normally used. A number of more advanced LE methods (for example Sarma's (1973) method and the Morgenstern-Price (1965) method) have since been developed which account for both force and moment equilibrium which improve the accuracy of the FoS calculation even further. For more information on these methods readers are directed to Abramson et al (2002).

Since the publication of Griffiths and Lane's (1999) paper adaptation of more advanced numerical methods for slope stability analysis has become common. Usually in these adaptations, the finite element method is combined with various schemes for strength reduction to arrive at an FoS or an estimate of the additional resistance to slope failure provided by the input soil parameters.

This article will examine the applicability of the finite element (FE) method to slopes and show how the results compare to the traditional LE approach. A study has been carried out in collaboration with Arup engineers and Oasys developers, comparing the results from LE and FE methods. Although this was initially intended to be a validation exercise it had some useful results which may provide guidance on when to consider using FE analysis, and in which circumstances the simpler and still reliable LE method is likely to be adequate.

The cases that have been examined are based on actual project data and published examples from Griffiths and Lane (1999), Chowdhury and Xu (2005) and Giam and Donald (1989).

2. Analysis methods

2.1 Limit equilibrium

Currently, most slope stability analyses involve LE analysis due to its simplicity and accuracy. These methods consist of cutting the slope into fine slices and applying appropriate equilibrium equations (equilibrium of the forces and/or moments). According to the assumptions made on the efforts between the slices and the equilibrium equations considered, many alternatives were proposed, such as the Bishop and Fellenius methods. In most cases, they are shown to give similar results. For example, Duncan (1996) reported that the difference between various methods was less than 6%. For this study, Oasys Slope, a limit equilibrium slope stability analysis program, was used. This offers a number of methods but, for each analysis, the Bishop method was applied for ease of comparison with the FE analysis.

2.2 Finite element analysis

As computer performance has improved, the application of FE in geotechnical analysis has become increasingly common. These methods have several advantages: to model slopes with a degree of very high realism (complex geometry, sequences of loading, presence of material for reinforcement, action of water, laws for complex soil behaviour) and to better visualise the deformations of soils in place. However, it is critical to understand the analysis output due to the larger number of variables offered to the engineer. Cases where severe failure has occurred, such as that of the Nicoll Highway, Singapore, highlight the importance of understanding the chosen numerical method and the failure criteria. See Puzrin et al, 2010 and Whittle and Davies 2006, for more information on the causes of the failure which included errors in the modelling work undertaken during the design and which ultimately resulted in the deaths of four people.

To analyse slopes, the strength reduction method is applied. This method is based on the reduction of the cohesion (c) and the tangent of the friction angle (tanφ) of the soil. The parameters are reduced in steps until the soil mass fails.

The study used Oasys Safe, a program for soil analysis by finite elements. When developing the strength reduction methodology to be applied in Safe, a comparison was made between three differing techniques.

For all techniques, an initialisation run for a given slope model was carried out and the strains and displacements obtained in that run set to zero for the subsequent FoS assessment. In the first method, an incremental strength reduction was applied to the elastic Mohr-Coulomb material whereby for each follow-on increment the same reduction in global strength was applied.

The second method involved specifying separate, independent model runs with revised material parameters corresponding to specific percentage reductions in material strength. The third method used a new feature in Safe, in which the program automatically applies the same strength reduction in successive analysis increments, but once failure is observed, reverts to the last converged increment and refines the strength reduction to obtain an estimate of FoS to an acceptable accuracy.

In this study the failure criterion was set to be displacement-related. Other finite element programs may use different criteria to establish when failure is occurring.

3. Case studies

3.1 Homogeneous slope with no foundation layer

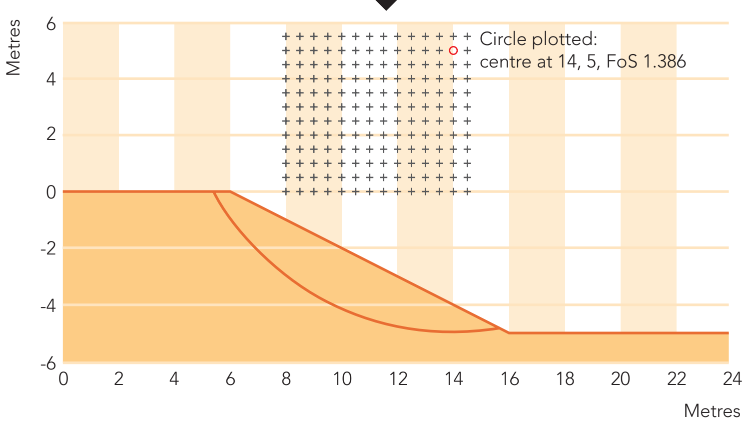
This example, based on example 1 from Griffiths and Lane's (1999) paper, corresponds to a homogeneous slope at a gradient of 1 vertical to 2 horizontal with an underlying high strength and stiffness layer (modelled as a fixed boundary in SAFE). The angle of internal friction of the soil is 20° and the cohesion is proportional to the unit weight of the soil and the height of the embankment.

Modelling was undertaken using the LE method and in SAFE using the three strength reduction methods outlined in section 2.2. The model geometry and results for the LE analysis are summarised in Figure 1.

Griffiths and Lane obtained failure at a factor of safety of 1.4, corresponding to a strength reduction of 28.6% (example 1 in their paper). Slope obtained a factor of safety of 1.386. This is very close to the result from Bishop & Morgenstern charts (1.380).

The three methods in Safe failed to converge at approximately the same strength reduction. This was at

FIGURE 1: LE ANALYSIS RESULTS FOR THE FIRST SLOPE GEOMETRY



the first increment or run with material strength less than 70% of the original parameters. The precise strength reduction or implied factor of safety therefore varied with the degree of strength reduction per iteration adopted. If the incremental change was relatively large, it would be easy to overpredict an FoS as a result.

The results for each method used are summarised in Table 1. These three approaches have potential implications for the results of modelling. For example, with an incremental strength reduction method (method 1), as a global FoS of 1 is approached there will be zones within the model where the yield criterion has been exceeded and the FoS is actually already < 1. This will then lead to a redistribution of stresses within the model away from the yielded zones and act to promote yielding at other points.

These yielded zones or points of weakness will be carried over to the next round of strength reduction and will to a greater or lesser extent influence the failure, making the solution path dependent. However, generating separate models with decreased strength parameters means that this behaviour will not occur within the model, which potentially has implications for the modelling of certain types of slope failure problem discussed below.

Where this behaviour is occurring then the

Table 1: Factor of safety results derived for the differing methods

Slope FoS	Safe Method 1: Incremental strength reduction of 5% in each increment above 1	Safe method 2 Independent runs with reduced strength parameters	Safe method 3 Strength reduced automatically corresponding to FoS in increments of 0.1, then 0.01 as failure approached
1.386	Increment 7: converged (70% strength) Increment 8 failed to converge (65% strength): deduced FoS: Greater than 1.428 but less than 1.538	Converged with 70% strength. Failed to converge with 69% strength. FoS = 1.45 approx	Failed to converge at FoS = 1.46 (68.45% strength)

incremental stress reduction could be thought of as a form of progressive softening/weakening behaviour akin to that which may occur due to strain softening behaviour triggered by shear or volumetric changes (for example seasonal pore pressure cycling leading to shrink-swell behaviour of the type described by Take and Bolton: 2011 and Leroueil: 2001)

In this work, the FoS results for the two methods seem to be identical, suggesting that this is not ultimately an issue for the constitutive model chosen. However, where some form of strain-softening constitutive model was in use or where it was important to model cyclic changes in pore water pressure response, this would become more significant (see for example Kovacevic et al, 2001; Nyambayo et al 2004; O'Brien, 2004; Scott et al 2007; and Rouainia et al 2009).

FIGURE 2: GEOMETRY FOR CLAY SLOPE WITH FOUNDATION LAYER – EXAMPLE A

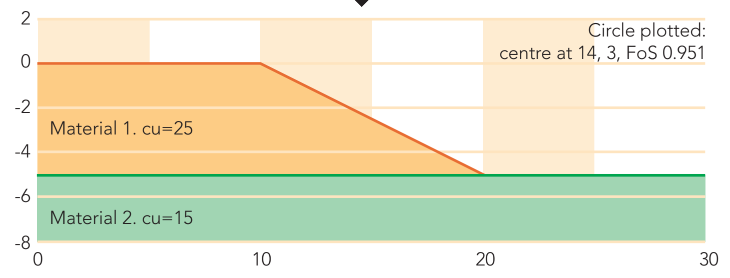


FIGURE 3: EXAMPLE C SLOPE RESULT – INITIAL ATTEMPT TO IDENTIFY CRITICAL SLIP SURFACE BY VARYING SLIP CIRCLE CENTRE AND RADII

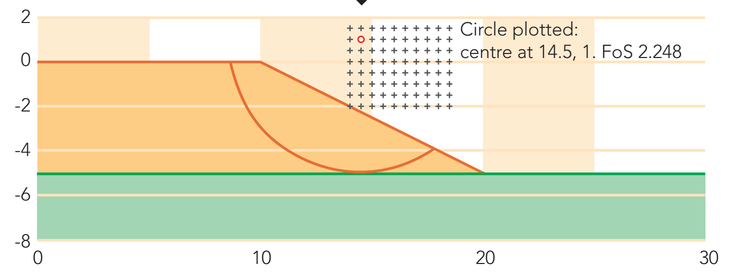
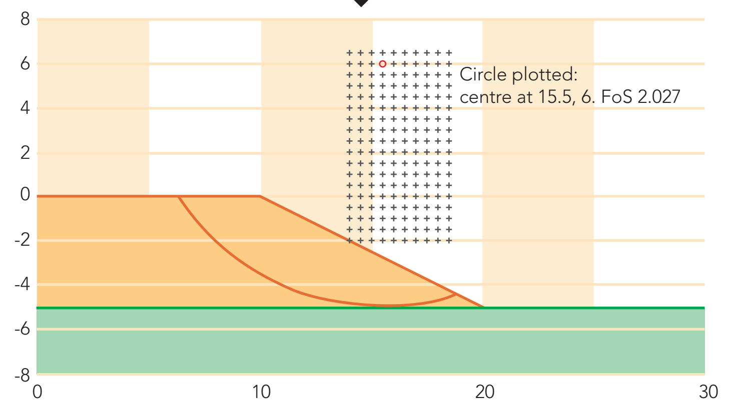


FIGURE 4: EXAMPLE C SLOPE RESULT – FAILURE SURFACE PRESUMED TO BE TANGENT TO STRATUM 2



3.2 An undrained clay slope with a foundation layer of different cohesion

This example models a homogeneous slope at a gradient of 1 vertical to 2 horizontal overlying a foundation layer. The angle of internal friction of the soil is 20° and the cohesion is proportional to the unit weight of the soil and the height of the embankment. The geometry is shown in Figure 2.

The ratio of the cohesion of the two layers is varied to produce a set of three analyses. This corresponds to Example 4 in Griffiths and Lane's (1999) paper.

With a weaker foundation layer, the failure mechanism is deep-seated. As the foundation layer is made stronger, this changes to a shallower failure through the toe of the slope. When the lower material has cohesion about 1.5 times that of the upper material, both mechanisms appear to be developing at the same time. Example A Cu1 = 25, Cu2 = 15; Example B Cu1 = 25, Cu2 = 37.5; Example C Cu1 = 25, Cu2 = 50.

Strength reduction methods could not be applied to Example A as it was already an unstable slope.

The results for Examples B and C illustrated the advantage of the finite element analysis in that an initial assumption about the location of the most likely failure surface does not have to be made by the user. In Slope, for example C in Table 2, an initial grid of circle centres obtained a higher factor of safety (Figure 3) than those published by Griffiths and Lane (1999). A lower FOS was obtained when the grid was extended using the automatic grid extension feature in the program (Figure 4). After considering the output from Safe (Figure 5) and adjusting the slip surface specification, a similar factor was obtained for a non-circular failure surface in Slope (Figure 6).

3.3 Stability analysis of cutting into multiple horizontal soil layers

In this example (taken from Chowdhury and Xu, 2005) a cutting into multiple horizontal soil strata is modelled. This example is a simplification based on a real cutting failure (the Congress Street Open Cut in Chicago). For further details the reader is referred to Ireland (1954). The problem geometry can be seen in Figure 7. The material properties are summarised in Table 3.

In this example the results from the FE and LE modelling demonstrate close agreement with a circular slip surface developing through soil layers 1 and 2, forming broadly tangent to the base of soil layer 2 and exiting the slope approximately 1.5 m above the toe. However, the FE modelling demonstrates that there is the possibility of a second slip surface forming within the steepened upper section of soil layer 1 (see the plot of shear strain in Figure 8) which might not have been

FIGURE 5: EXAMPLE C SAFE – 45% STRENGTH FAILED TO CONVERGE

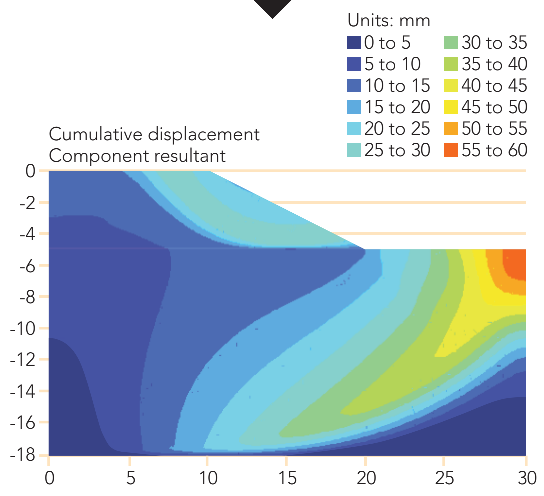


FIGURE 7: PROBLEM GEOMETRY AND FIXITIES (after Chowdhury & Xu, 2005)

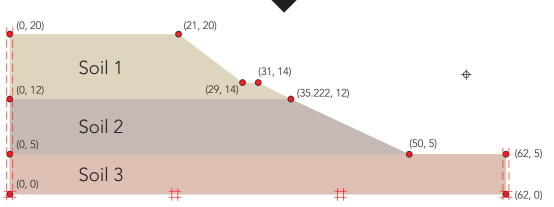


FIGURE 8: CRITICAL SLIP SURFACES LOCATED USING FE MODELLING. (Note presence of second smaller slip surface in upper layer)

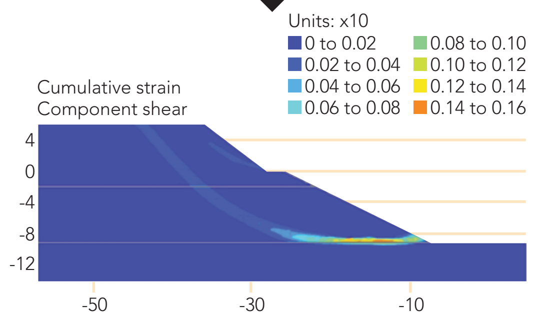


Table 4: FoS results for the differing software packages compared to the literature example

Chowdhury and Xu	Slope	Safe
FoS = 1.16	FoS = 1.21	FoS = 1.20

Table 2: Results for Slope and Safe analyses

Cu2/Cu1	Slope FoS	Safe Method: Incremental strength reduction of 5% in each increment above 1
0.6 Example A	0.951	n/a
1.5 Example B	2.042 (toe mechanism) 2.060 (deep failure)	2.0 (50% strength failed to converge)
2.0 Example C	2.025 (toe mechanism, circular) 2.127 (non-circular)	2.22 (45% strength failed to converge)

Table 3: Material parameters as used in the model (after Chowdhury and Xu, 2005)

Material	Young's Modulus (Pa)	Poisson's Ratio	Cohesion (kPa)	Friction Angle (°)	Density (kN/m³)
Soil 1 (Sand)	1 x 105	0.3	3	30	21
Soil 2 (Clay)	1 x 105	0.3	22	11	22
Soil 3 (Clay)	1 x 105	0.3	25	20	22

FIGURE 6: EXAMPLE C SLOPE RESULT – NON CIRCULAR SLIP

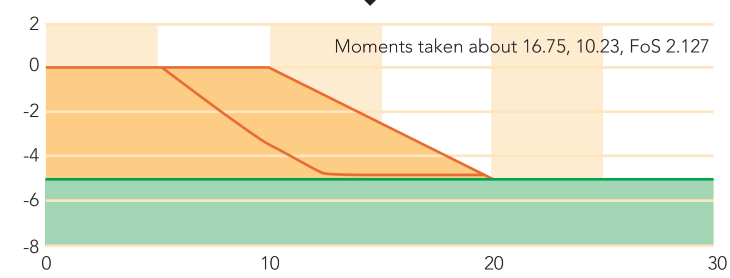
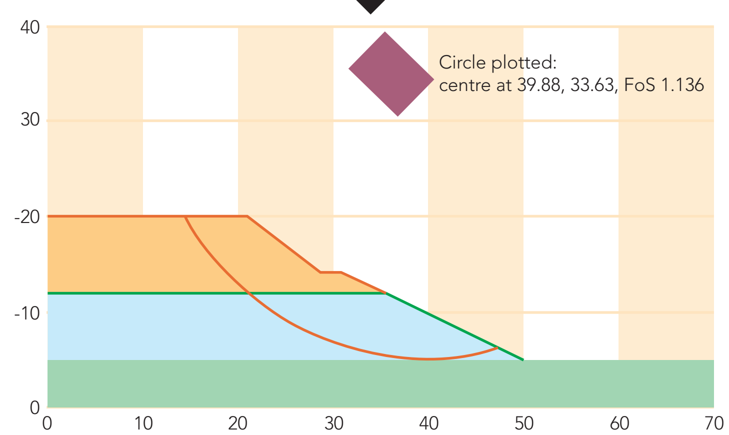


FIGURE 9: CRITICAL SLIP SURFACE FOUND BY LE METHOD IN SLOPE. (Note that this technique failed to identify potential occurence of the second smaller slip surface seen in the FE modelling)



located by the user undertaking a search for the lowest FOS slip surface in Slope (Figure 9).

The derived FoS values are summarised in Table 4 where it can be seen that Safe and Slope show good agreement.

3.4 Stability analysis of a slope with a sub-horizontal weak band and varying phreatic surface

This problem is a variation of an example provided by Giam & Donald (1989) with a slope incorporating a thin, steeply dipping weak soil layer and a variable water table. The original example included vertical loading on the crest of the slope and a cohesionless low strength layer which when analysed resulted in an FoS of less than 1.0. The FE program is not able to find an FoS less than 1.0 as the soil mass will fail instantly before any iterations of the strength reduction FoS search can be undertaken. As such the loading from the crest was removed and the strength of the weak band was increased to create an initially stable slope. The problem geometry, phreatic surface, strata and fixities are summarised in Figure 10. The material parameters used in this model are shown in Table 5.

Initial assessment of the FoS was undertaken in Oasys Slope assuming a circular slip surface and allowing an automated search for a circular slip with variable centres and slip radii. The resultant minimum FoS for this method was calculated as 1.36. The slip centre and radius can be seen in Figure 11. This was then repeated assuming that the circular slip would form tangent to the base of the low strength layer. The resultant slip from this analysis is shown in Figure 10 with a slightly lower FoS of 1.35.

The results of the FE analysis in Oasys Safe indicate that a non-circular slip forms within the low strength layer and parallel to its base with the slip surface day lighting at the toe of the slope and approximately 8m back from the break of slope along the crest (see Figure 12). The Safe FoS Phi'/C' reduction method gives FoS values of 1.18. These values are 15% lower than the FoS calculated initially in Slope assuming a circular slip surface.

FIGURE 10: GEOMETRY OF SLOPE WITH A SUB-HORIZONTAL WEAK BAND AND VARYING PHREATIC SURFACE SHOWING SOIL STRATA, GEOMETRY CO-ORDINATES AND FIXITIES

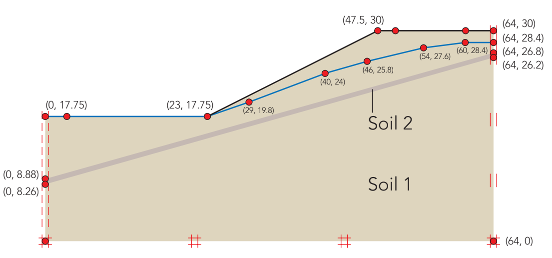


FIGURE 11: LOWEST FOS CIRCULAR SLIP SURFACE ASSUMING SLIP FORMS TANGENT TO BASE OF LOW STRENGTH STRATA

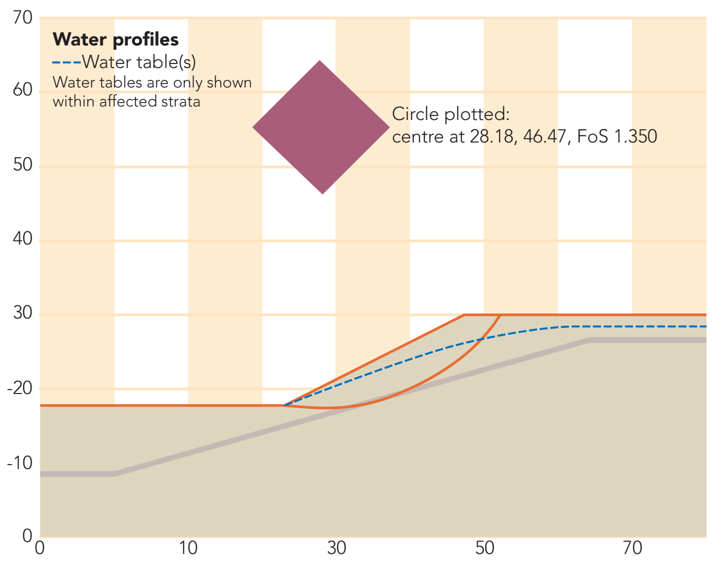


Table 5: Material parameters as used in the model (after Giam & Donald, 1989)

Material	Young's Modulus (Pa)	Poisson's Ratio	Cohesion (kPa)	Friction Angle (°)	Density (kN/m³)
Soil 1	1 x 105	0.3	20.0	28.5	18.84
Soil 2	1 x 105	0.3	10.0	15.0	18.84

The Safe output indicates a maximum displacement magnitude of 12mm (not shown here) whereas the results of a Plaxis analysis show that the maximum displacement in this case is approximately 3m (Figure 13). This illustrates that an FE user should be aware that the absolute values of post failure deformation are meaningless and that it is the information provided by FE modelling on the geometry of the slip wedge or circle and the location of the potential slip surface that is of primary interest.

In this case the slip surface geometry was extracted from the FE results and used to specify a non-circular slip surface to undertake an analysis in Slope. The co-ordinates of the slip surface are shown in Table 6. The analysis results are displayed in Figure 14 (overleaf) where it can be seen that the calculated FoS is 1.181. This very closely matches the value computed by Safe.

The example modelling undertaken above strongly suggests that for slope stability problems where there is subsurface heterogeneity with materials of contrasting strength/stiffness or with inclined strata (most likely to occur in cuttings as opposed to constructed embankments), the critical slip surface is unlikely to be circular and thus normal slope stability assessment using limit equilibrium software will not capture the likely geometry of the slip surface and will to a greater or lesser extent overestimate the FoS of the problem being modelled, as such FE modelling may be a more appropriate tool in these situations.

FIGURE 12: SAFE PLOT OF SHEAR STRAIN HIGHLIGHTING THE NON-CIRCULAR NATURE OF THE SLIP SURFACE

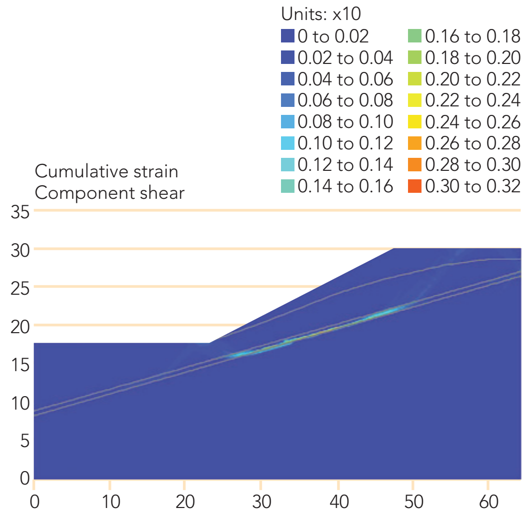


FIGURE 13: PLAXIS DISPLACEMENT CONTOUR PLOT

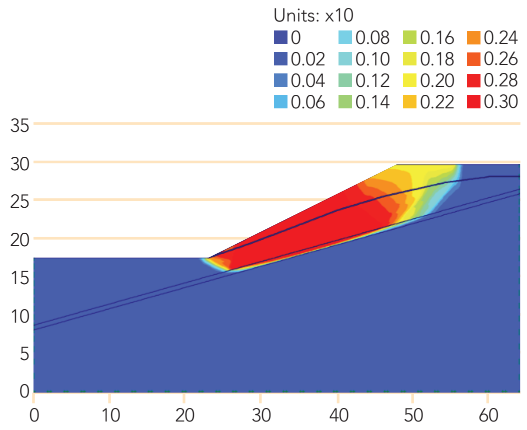


Table 6: Coordinates for non-circular slip surface used in final slope model derived from FE modelling.

[x]	[y]
22.69	17.75
26.00	16.25
27.90	16.20
46.00	21.20
49.80	22.80
56.00	30.00

The close match between the FoS values calculated by the FE software and the LE software when a non-circular slip surface is derived based on the FE results demonstrates that the LE methodology is still valid. However, at present the input of non-circular slip surfaces relies on engineering judgement and is a manual process.

4. Summary of results

Through the application of different methods of analysis to a range o f cases, the following conclusions have been drawn by the authors.

First, although Limit Equilibrium methods have been in use from the early 20th century, the FoS obtained are shown to compare very well to those obtained by FE analysis. However, it is shown that for more complex problems FE can better demonstrate the geometry of failure surfaces.

The example provided by Chowdhury and Xu (2005) demonstrated how a second slip surface forming within the steepened upper section was immediately visible in the FE analysis. When using LE, the larger slip surface is immediately evident but the second slip surface would only be found by examining other slip surfaces. This requires engineering judgement and if this was missed, the reinforcement design may not be sufficient, potentially having significant consequences for stability.

The slope analyses based on Giam & Donald (1989), shows that for more complex stratigraphies where the critical slip is unlikely to be circular, typical LE analysis will overestimate the FoS. The non-circular slip surface LE method (Janbu) does produce comparable results to FE but this requires a level of judgement from the design engineer and would require a large number of manual iterations to attempt to estimate the most critical non-circular geometry, which could take significant additional time and may ultimately prove unsuccessful.

5. Conclusion

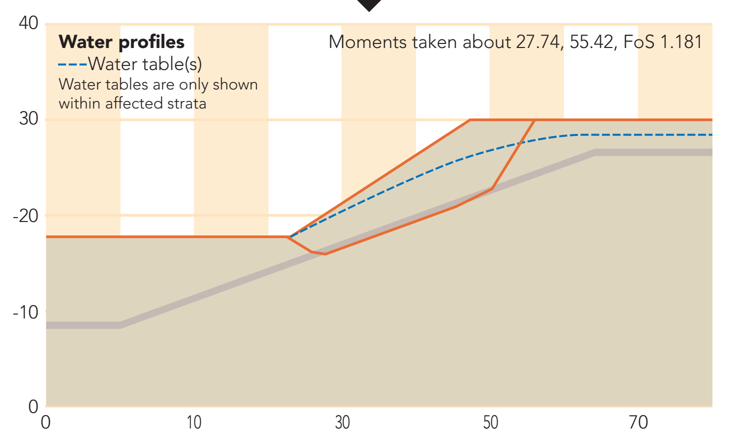
As computers and their application evolve in geotechnical analysis, it seems that we should be looking to more advanced ways to analyse slope stability. This study has shown that there are significant opportunities in using the more comprehensive finite element analysis. However, the traditional Limit Equilibrium method remains able to produce accurate and reliable results.

To return to our initial question, "Slope stability analysis – limit equilibrium or the finite element method?" the answer would appear to be that both have their advantages and disadvantages with the choice of which method to use depending on some of the considerations described below:

The method the user selects should be based on the complexity of the problem to be modelled. For example problems with complex geometries or that require analysis of seepage, consolidation and other coupled hydrological and mechanical behaviour (pore water pressure induced shrink swell cycles for example) along with those problems with more complex mechanical soil responses (eg post failure strain softening and progressive failure) may be better tackled using FE analysis. Conversely, simpler problem geometries or where complex material responses are not expected, or those problems where data is limited or it is necessary to make an initial stability estimate before undertaking more complex analysis may better be undertaken in LE software such as Slope.

In either case, as highlighted by the Nicoll Highway failure and the examples above, it is important that the user fully understands the assumptions inherent in the chosen modelling method when interpreting the results and applying them in any potential new slope design or existing slope stability assessment.

FIGURE 14: RESULTS FOR THE NON-CIRCULAR LE SLOPE MODEL WITH SLIP GEOMETRY DERIVED FROM FE MODELLING



References

Abramson L W; Lee T S; Sharma S; Boyce G M (2002). Slope Stability Concepts. Slope Stabilisation and Stabilisation Methods, Second Edition, published by John Wiley & Sons, Inc., pp. 329-461

Bishop A W (1955). "The use of slip circles in stability analysis of slopes". Géotechnique, 5:7-17.

Chowdhury R N; Xu D W (1995). "Geotechnical system reliability of slopes". Reliability Engineering and System Safety, 47:141-151

Duncan J M (1996). "State of the Art: Limit Equilibrium and Finite-Element Analysis of Slopes", Journal of Geotechnical Engineering, 122:577-596

Fellunius W (1936). "Calculations of the Stability of Earth Dams". Proceedings of the Second Congress of Large Dams. 4:445-63, Washington DC

Giam P S K; Donald I B (1989). Example problems for testing soil slope stability programs. Civil Engineering Research Report No. 8/1989, Monash University

Griffiths D V; Lane P A (1999). "Slope stability analysis by finite elements". Géotechnique, 49:387-403

Ireland H O (1954). "Stability analysis of the Congress Street open cut in Chicago". Géotechnique, 4:163-168

Kovacevic K; Potts D M; Vaughan P R (2001). "Progressive failure in clay embankments due to seasonal climate changes". Proceedings of the 5th International Conference on Soil Mechanics and Geotechnical Engineering, Istanbul, 3:2127-2130

Leroueil S (2001). "Natural slopes and cuts: movement and failure mechanisms". Géotechnique 51:197-243

Morgenstern N R; Price V E (1965). "The Analysis of the Stability of General Slip Surfaces". Geotechnique, 15:77-93

Nyambayo V P; Potts D M; Addenbrooke T I (2004). "The influence of permeability on the stability of embankments experiencing seasonal cyclic pore water pressure changes". Advances in Geotechnical Engineering: Proceedings of the Skempton Conference (Jardine R J; Potts D M; Higgins K G (eds)). Thomas Telford, London, 2898-2910

O'Brien A (2004). "Old railway embankment clay fill: laboratory experiments, numerical modelling and field behaviour". Advances in Geotechnical Engineering: Proceedings of the Skempton Conference (Jardine R J; Potts D M; Higgins K G (eds)). Thomas Telford, London, 2:911–921

Puzrin A M; Alonso E E; Pinyol N M (2010). Geomechanics of Failures, Springer Dordrecht.

Rouainia M; Davies O; O'Brien T; Glendinning S (2009). "Numerical modelling of climate effects on slope stability". Proceedings of the Institution of Civil Engineers, Engineering Sustainability, 162:81-89

Sarma S K (1973). "Stability Analysis of Embankment and Slopes". Géotechnique, 23:423-33

Scott J; Loveridge, F; O'Brien, A S (2007). "Influence of climate and vegetation on railway embankments". Proceedings of ECSMGE, Madrid: 659-664

Take W A; Bolton M D (2011). "Seasonal ratcheting and softening in clay slopes, leading to first-time failure". Géotechnique, 61:757-769

Whittle A J; Davies R V (2006). "Nicoll Highway Collapse: Evaluation of Geotechnical Factors Affecting Design of Excavation Support System". International Conference on Deep Excavations, 28-30 June 2006, Singapore