Seismic earth pressures – a technical note

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This brief technical note summarises the key methods in codes and the literature and how they are applied. It is a synthesis of papers presented by the author last year at the New Zealand Geotechnical Symposium [1] [2] and recent experience on projects in Europe. The focus is on pseudo-static earth pressures used in typical day-to-day engineering practice, where the transient dynamic load of an earthquake is simply represented by an additional static pressure or force resultant on the structure being designed. The use of more advanced methods to consider the dynamic response is only discussed briefly where understanding the dynamic soil-structure interaction may be important, such as deriving the effective design inertia.

Seismic design

Philosophy
For static design, the aim of code guidance is to ensure that catastrophic collapse or failure is avoided (Ultimate Limit State or ULS), but also to ensure that the performance of the structure under the applied load will not be disappointing in terms of wall movements – be they settlements or deflections (Serviceability Limit State or SLS). For the seismic case we wish to check essentially the same limit states for a “design” earthquake. The design earthquake will have been specified by national or industry code, or a special study to determine the acceptable level of hazard for the structure. This increasingly considers different levels of seismic hazard to ensure the performance at frequent earthquakes is not disappointing (analogous to an SLS check) as well as ensuring rare earthquakes do not lead to failure (a ULS check). Here “failure” may be defined in terms of unacceptable wall displacements rather than a catastrophic failure of the wall. A clear understanding of the required performance of the structure in terms of displacements is advisable prior to undertaking design. Modern codes (e.g. Eurocode 8 (EC8) [3]) may implement this philosophy implicitly by assuming performance during frequent events will be sufficient for normal structures if the ULS check performed for rare events is satisfied with a given safety margin. Facilities supporting hazardous materials and processes (e.g. LNG, Nuclear) tend to have more explicit requirements for the respective hazard levels – the latter will have usually

Figure 1. Typical loading diagram for a gravity quay wall under seismic loading, assuming active earth pressure conditions occur at the ULS.
been derived from a special study carried out by Engineering Seismologists for these types of structures.

**Methodology**

The design methodology has been expounded by Steedman \[4\] in a paper on seismic design of retaining structures. In addition, useful guidance, particularly in relation to the performance based design framework may be obtained from PIANC \[5\] and to a lesser extent from EC8. A typical loading diagram for a quay wall under pseudostatic loading is shown in Figure 1. Inertia loads on the retaining structure should also be considered in addition to inertia load on the soil and water.

**Dynamic earth pressure theories**

The commonly cited methods to assess seismic earth pressures (or “dynamic earth pressures”) in codes and texts make basic assumptions about how the wall and soil interact, or together referred to as the “wall-soil-system”. These fall into the two extremes of system response: perfectly rigid or no deflection or displacement; and walls free to displace and/or deflect until minimum (active) earth pressures occur. Selection of the appropriate method will depend on the structure being considered, and the definition of the ULS for the particular structure. The bulk of this paper is geared to describe these two basic methods, their

\[
\begin{align*}
K_{AE} &= \frac{\sin^2(\psi + \phi - \theta)}{\cos(\theta) \sin^2(\psi) \sin(\psi - \theta - \delta)(1 + X)^2} \\
K_{PE} &= \frac{\sin^2(\psi + \phi - \theta)}{\cos(\theta) \sin^2(\psi) \sin(\psi + \theta)(1 - Y)^2}
\end{align*}
\]

where \( X = 0 \) if \( \beta > \phi - \theta \), otherwise: \( X = \left( \frac{\sin(\phi + \delta) \sin(\phi - \beta - \theta)}{\sin(\psi - \theta - \delta) \sin(\psi + \beta)} \right)^{0.5} \)

where \( Y = 0 \) if \( \theta > \beta + \phi \), otherwise: \( Y = \left( \frac{\sin(\phi) \sin(\phi + \beta - \theta)}{\sin(\psi + \beta) \sin(\psi + \theta)} \right)^{0.5} \)

\( \gamma_{ratio} = 1.6 \) for dynamically pervious backfill, 2.0 for impervious backfill, 1.0 for dry. Ref. Matsuzawa et al. \[7\]

Figure 2. Mononobe–Okabe dynamic earth pressure coefficient calculation; (a) active and (b) passive limit conditions. Coefficient \( K \) is the ratio of lateral to vertical effective stress at the wall-backfill interface. NB: The passive condition has no wall friction \( \delta \) considered, after EN 1998 \[3\].
assumptions and limitations. It also presents alternative approaches and guidance from the literature where they are required.

**Rigid walls free to displace to active-earth conditions**

**Mononobe-Okabe method**

The dynamic earth pressures at limit equilibrium may be estimated using the classic “Mononobe-Okabe” (M-O) [6] [7] earth pressure method, where the applied inertia results in rotation of the principal stress (angle $\theta$ in Figure 2), enlarging the Coulomb active wedge (or decreasing for the passive case) at limiting equilibrium. The method enables calculation of the lateral earth pressure coefficient $K$, which is a ratio of horizontal to vertical earth pressure at the wall-backfill interface. In general, M-O provides a relatively good estimate (cf. model testing), provided the assumptions are met; chiefly that the wall is able to deflect or displace away from the soil to the minimum limit condition – thus mobilising full shear on the failure plane; secondly, the soil is rigid such that the acceleration applied is uniform; and thirdly, the soil is cohesionless and dry [8].

An important point is that the original method does not make any claim to where the point of action of the resultant force should be applied to the wall, an important consideration for assessing overturning stability.

These assumptions and limitations affect the applicability of the M-O method and raise a number of issues that will be addressed to some extent within the remainder of the paper. These include:

- the “Coulomb error” for passive earth pressures with wall friction applied;
- walls that do not displace sufficiently to form an active wedge;
- the appropriate design acceleration coefficient to apply to the wall backfill;
- the point of action of dynamic thrust;
- cohesive soils; and
- water pressure effects.

The longevity of the M-O method is due in part to its simplicity but also its adaptability as various modifications have been suggested by researchers over subsequent years and eventually adopted in code guidance. It is also however much abused and is often applied to situations which it was never intended – such as rigid walls, tied back and stiff cantilever embedded walls. Care should therefore be taken in its adoption to avoid gross errors.

**Passive earth pressure error**

An important caveat with M-O for passive pressures with wall friction ratio $\delta/\phi > \frac{1}{2}$ is that an unconservative error develops, inherited from Coulomb’s passive earth pressure equation which assumes a linear failure plane – in reality it is curved [10]. Because of this, EC8 ignores wall friction en-

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tirely. An alternative to remove this slight conservatism for both static and seismic case is to adopt a log-spiral shaped failure plane [11], refer Figure 3. Note that passive earth pressure requires a significant amount of wall deflection to be mobilised, which may well exceed the ULS criteria for the wall. Reference to typical wall movements required to mobilise the active and passive earth pressure conditions is provided in Eurocode 7 (EC7) [12], and NAVFAC [13]. A factor should be applied to limit the mobilisation of passive earth pressure for most design situations where the wall is embedded in soil.

Large design inertia
The M-O method becomes unstable when the sum of interia angle $\theta$ and backfill slope angle $\beta$ exceed the angle of shearing resistance of the soil, $\phi'$. In this case, the square-root term on the denominator of the equation becomes complex, and cannot be solved. Matsuzawa et al. [9] proposed a simplification to avoid this problem which is adopted by EC8 and is included in Figure 2. However, this results in very large active wedge angles, which may be unrealistic in reality. The recommended approach to avoid this problem is to consider the critical acceleration at which the wall will begin to displace (i.e. factor of safety $= 1$), and carry out a performance based assessment of the retaining structure (more discussion is provided on this aspect in a subsequent section on displacement estimation).

An alternative approach is to consider the modification of the M-O method by Koseki et al. [14], which considers the punctuated development of multiple wedges:

1. The initial active wedge occurs under static conditions with peak soil strength ($\phi'_{\text{peak}}$) and no inertia applied ($k_h = 0$). Shearing continues along the pre-defined wedge until residual strengths ($\phi'_{\text{res}}$) are developed on the failure plane. The earth pressure increases, and this is used for static design.

2. If the active wedge has not developed under static conditions, it may do so when moderate earthquake inertia is applied to the same wedge geometry as 1. They recommend a value of $0.2g$ for typical design as the critical value at which the wedge forms (using M-O approach). Again, lateral earth pressures are considered using a wedge geometry based on peak strengths, but with a reduction to residual strengths.

3. The peak earthquake inertia above this critical value causes a secondary larger wedge to develop with peak strengths. This will cause larger earth pressures to act on the wall.

The approach has been referenced by the ISO draft code on performance based design in earthquake geotechnical engineering [15], and Japanese Geocode 21 [16]. The main advantage over M-O is that consideration is made to the development of active wedge prior to the earthquake occurring, and consideration of the effect of strain softening.

Figure 4: Damage to quay wall at Derince, Leman industrial facility. Kocaeli, Turkey Earthquake of 17/8/99. Image courtesy Earthquake Engineering Field Investigation Team (EEFIT), UK [24].
from peak to residual along the pre-defined wedge. For large events, the peak strength of the soil may be considered in the formation of a new active wedge, which assists in avoiding the problems with M-O and large inertia.

**Point of action of dynamic thrust**

From studies in the literature three interacting components have been identified to affect the point of action of the dynamic earth pressure resultant ("dynamic thrust"). These are:

- Ground motion frequency
- Wall-soil system relative flexibility
- Global movement of the wall-soil system.

Seed and Whitman [8] divided the M-O earth pressure resultant ($P_{AE}$) into separate static ($P_A$) and incremental dynamic ($\Delta P_{AE}$) components $P_{AE} = P_A + \Delta P_{AE}$, and recommended based on model testing, that the incremental dynamic component be considered to have a point of action at 0.6$H$ measured from the base ($H$ = wall height). Often this advice was simplified by adopting an inverted triangle for the dynamic earth pressure profile. Steedman and Zeng [17] in a pseudodynamic analysis of the active wedge showed the point of action was a function of the height of the wall, the shear wave velocity of the backfill, and the period of the ground motion; essentially showing that the point of action of $\Delta P_{AE}$ is dependent on the proportion of inertia that affects the upper third of the assumed active wedge, where the bulk of the mass resides. Thus the 0.6$H$ of Seed and Whitman was at best an upper-bound and would be conservative for design purposes, particularly in a performance-based framework.

Veletos & Younan's [18] dynamic analysis of fixed base walls considers a visco-elastic medium without a pre-defined wedge. Their work provides the point of action of the dynamic thrust for varying wall-soil system flexibilities from Wood's ~0.6$H$ for perfectly rigid wall [19], down to less than $\frac{1}{2}H$ for very flexible walls. Richards et al. [20] investigated the mode of wall movement and showed that rotation about the base caused the point of action to drop to the lower $\frac{1}{2}$, whilst for a translation mode it was at 0.5$H$, and for rotation about the upper portion of the wall 0.67$H$.

It is perhaps in light of this work from the preceding decade that EC8 is the first modern code to depart from the 0.6$H$ or inverted triangle convention and recommends applying the point of action of the dynamic component at mid-height or 0.33$H$ if free to rotate about the toe.

**Cohesive soils**

A number of methods have been adopted to consider cohesive soils in the literature. Whitman [21] refers to some with a degree of scepticism. Recently Anderson et al. [22] consider soils with significant $c'$ and $\phi'$ using limit equilibrium slope stability software to determine the dynamic earth pressure coefficient $K_{AE}$, and provide charts for practical use. For the undrained case ($\phi' = 0$), the use of the same approach or a trial wedge method may be adopted to determine the dynamic earth pressure. The Japanese Ports and Harbours design manual [23] provides an equation for determining the undrained dynamic earth pressure. Consideration should be given to reduction in the shear strength due to cyclic loading and generation of excess pore pressures concurrent with the application of peak dynamic loading. An alternative is to consider an effective stress based approach using a modified M-O method, which is discussed in the subsequent section.

**Dynamic water pressures**

For many retaining structures in terrestrial environments, walls are designed with drainage to ensure the build up of static water pressures does not occur, however this is not possible for walls permanently below the water table – be they basements, underground structures, or marine structures such as quay walls. Here the presence of a permanent static water table has a significant effect on wall stability. During earthquakes this has three important effects:

1. The weight vector in the M-O active wedge (refer Figure 1) is almost halved due to buoyancy, thereby greatly increasing angle of the weight vector $\theta$. This effect was reported by Matsuzawa et al. [9], who noted that for free draining conditions during cyclic loading (referred to as "dynamically pervious", such that excess pore pressure generation under cyclic loading is minimal), this factors the effect of horizontal inertia by approximately 1.6 (dry unit weight/unit weight of water). For "dynamically constrained" conditions (i.e. fine grained deposits that during dynamic loading will result in essentially undrained behaviour), the factor is around 2.0 to 2.2, depending on the ratio of saturated unit weight to unit weight of water. It is recommended that care is taken to derive the saturated and dry weights from soil phase relationships so that this ratio is correctly estimated as the magnitude of the factor can have a significant effect on the estimated dynamic pressures.

2. If the soil remains dynamically pervious – e.g. open or coarse granular fills that allow the free flow of water between grains, dynamic water pressures should also be considered. The method of Westergaard [25], developed for free water bodies such as dam reservoirs, is adapted for the presence of soil grains – once again reference is made to [9] for guidance on this assessment; it is also a useful guide as to whether the material will behave as "dynamically pervious" or "dynamically constrained". EC8 ignores the relative effect of soil grains on the dynamic water pressures in the backfill, and it is true that the effect is generally relatively minor.

3. If the soil is dynamically constrained, a degree of excess pore pressures will be generated during cyclic loading, as with each successive cycle of loading pore pressures generated due to volumetric changes on shearing can
not dissipate in time before the subsequent cycle of shearing occurs. The effective stress path will thus progressively work its way towards the failure envelope, or phrased in total stress terms, the shear strength degrades progressively with each subsequent cycle. The main concern in this instance is for soils whose steady state shear strength – that is, the strength of the soil after large shear strains have taken place – is less than the in-situ strength, resulting in complete collapse of the deposit (flow liquefaction). Most catastrophic failures of quay walls during earthquakes are due to this problem occurring in hydraulic fills (e.g. Port Island, Kobe 1995, Derince Port, Turkey 1999 – refer Figure 4). Preventing this problem should be the first priority of the designer, principally through the use of ground improvement techniques such as stone/vibro columns. The condition where generation of excess pore pressures build up progressively but do not necessarily lead to liquefaction, should also be considered as strength is lost from the deposit, and dynamic earth pressures will be larger. One mitigating effect is that earthquake energy is consumed in order to shear the deposit, and by the time significant pore pressures are generated, the peak loading cycles may have already occurred. The guidance to consider excess pore pressure ratio $r_u$ in the calculation of angle $\theta$ is provided by [10] as a modification to reference [9]. The estimation of $r_u$ for a given deposit and level of earthquake shaking is beyond the scope of this paper.

**Elastically constrained rigid walls**

A rigid wall will typically not deflect sufficiently to develop an active or passive failure wedge. Thus most codes recommend Wood’s [19] solution (e.g. Eurocode 8 (EC8) [3]) for an elastically constrained rigid wall:

$$\Delta P_E = \gamma H^2 k_h F_p ,$$

where $F_p$ is a dimensionless thrust factor and a function of the stiffness of the system, and $H$ the height of the wall. For typical soils $F_p$ is approximately unity, and is what EC8 assumes. This may be thought of as a uniform inertia $k_h$ applied to a rigid block of soil of dimensions $H \times H$. The point of action for this force resultant was determined to be $0.58H$, sometimes simplified to $0.6H$. To determine the dynamic earth pressure profile, the recommendation of Matthewson et al. [26] is often adopted, where the dynamic earth pressure decreases linearly from the top of the wall at $1.5k_h\gamma H$ to the base at $0.5k_h\gamma H$ – refer Figure 5. The static earth pressure component is based on at-rest pressures or compaction pressures where the wall has backfill present. By mere observation it is clear the dynamic load will be significantly greater than the M-O method, and for many engineering problems will be very conservative.

**Fixed base flexible walls**

It may be appreciated that there are many cases when neither of the base assumptions recommended by codes (i.e. M-O and Wood) are strictly applicable, in which case further analysis beyond the basic code methods may be required. Veletsos and Younan [18] have shown Equation (1) to be valid for rigidly elastic wall-soil conditions, but if the wall-backfill, and/or wall-base are more flexible, the dynamic earth pressures can be considerably less. They provide tabulated $F_p$ factors for varying soil-wall and soil-foundation flexibilities, demonstrating Wood’s solution for rigid systems, through to typical wall flexibilities ($F_p = 0.5$) and down to very flexible systems where $F_p = M-O$ values of $\Delta K_{AE}$ (i.e. $K_{AE} - K_A$, where the latter is based on the Coulomb equation). Moreover Psarropoulos et al. [27]...
showed their results compared well to FE modelling. US Army Corps Engineers [28] have adopted this method for the structural design of L-wall stems.

### Basement walls and tunnels

For basement walls, Ostadan & White [29] (see also Ostadan [30]) developed a simplified method, based on 1-D site response analysis (such as using SHAKE [31], Oasys SIREN [32], or a similar program) following which a response spectrum analysis is performed to derive the maximum dynamic earth pressure. The final profile is obtained using a semi-empirical normalised earth pressure curve, based on a series of dynamic soil structure interaction (DSSI) analyses. Their results showed that depending on input ground motion, soil and wall properties, the M-O and Wood solutions provided lower and upper bounds respectively. Thus over-predictions by using Wood may be assessed and reduced through this approach that considers the dynamic wall-soil system response to the design ground motion inherently. Figure 6 shows a comparison between methods to derive the dynamic earth pressure profile that was carried out for a typical project, including common interpretations of the use of M-O. This method has been referenced by FEMA 450 [33].

A displacement based design approach to assess racking effects in box tunnels, by Wang [34], is based on a similar approach – parametric DSSI analyses have been performed to derive a method to estimate the displacement and hence strains induced in the tunnel structure. This approach avoids the problem of estimating dynamic earth pressures by either the M-O or Wood methods, which would be inappropriate for most tunnel structures.

An alternative pragmatic design approach is to use a combination of 1-D site response analysis to obtain free field ground displacements, in combination with a pseudostatic soil-structure interaction analysis performed in 2-D finite element analysis software such as Plaxis, Oasys SAFE, Abaqus, FLAC etc., by a means of prescribed displacements (e.g. Free et al. [35]). Further discussion on the seismic analysis of underground structures is provided by

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**Figure 6.** Comparison of calculated dynamic earth pressures for a rigid embedded wall, founded on rock. Sand with $\phi'=35^\circ$, $D_r=65\%$. PGA (rock) = 0.15g. PGA (soil) = 0.2g (UBC97), PGA (soil) =0.27g (SHAKE). NB: M-O interpretations for comparison only, not strictly applicable for this wall type.
Hashash et al. [36]. Kontoe et al. [37] note that such simplified methods often provide reasonable results despite the inherent simplifications.

**Design inertia, phase effects and amplification**

**Effective acceleration**

Sarma and Yang [38] attempted to add a theoretical basis to the common practice of applying a factor of \( \frac{1}{2} \) or \( \frac{1}{3} \) to Peak Ground Acceleration (PGA) by considering the acceleration required to generate 95% of the “energy” in an earthquake record (measured in Arias Intensity), dubbed the \( A_{95} \) parameter. From the results of a study of 135 earthquake recordings, a best fit correlation of \( A_{95} = 0.675 \times \text{PGA} \) was obtained. In contrast, Japanese practice adopts the proposal by Noda et al. [39] where \( k_h \) is determined as follows:

\[
k_h = \begin{cases} 
\frac{1}{3} (\text{PGA})^{\frac{2}{3}} & \text{if } \text{PGA} \geq 0.2g, \\
\frac{1}{2} (\text{PGA})^{\frac{1}{2}} & \text{if } \text{PGA} < 0.2g.
\end{cases}
\]

This is based on back-analysed estimates of \( k_h \) from quay wall performance during earthquakes, and forms an upper-bound estimate, whilst a mean value is around \( 0.6 \times \text{PGA} \) (PIANC [5]). Al Atik and Sitar [40] found that a value of \( 0.65 \times \text{PGA} \) provided reasonable estimates for matching the use of M-O earth pressures centrifuge test results of embedded walls. The actual value will depend on the ground motion, wall geometry, soil properties, and whether liquefaction occurred. None of these empirical approaches considers all of these factors systematically.

Steedman and Zeng [17] investigated the assumption of infinite stiffness on dynamic active earth pressures through a pseudo-dynamic analysis method. Their results suggest phase effects on earth pressures are of small significance, but the effects of amplification were significant, and clearly both these effects would be more pronounced with large walls where peak ground velocity (PGV) rather than PGA will control the maximum inertia applied to the wall during the design earthquake.

EC8 is perhaps the first code to recommend that walls larger than 10m be considered in a site response analysis for design. Arguably this should be performed as a 2-D site response analysis that may consider the change in profile of the ground surface provided by the wall, account for the relative stiffness differences between wall and backfill, and the dynamic response of any structures present that may influence the wall behaviour. A 1-D analysis cannot capture these important aspects. Recently Anderson et al. [22] presented the results of a parametric study to consider these effects more systematically using QUAD4M – an equivalent linear 2-D site response program [41]. They produced a chart showing significant reduction in design ground motion from PGA may be taken depending on the height of the wall, and the ground motion characteristics – the latter simplified as a ratio of the design spectra at a period of 1s and at PGA (i.e. relative contribution of long period to short period motion).

**Displacement estimation**

The most common method used to reduce the design acceleration from PGA, is to utilise the Newmark sliding block concept [42], originally developed for estimating co-seismic displacements of non-catastrophic embankment failures. Richards and Elms [43] adopted this method for co-seismic sliding of retaining structures in an early application of the performance based design concept. EC8 considers this method inherently in the analysis procedure, by allowing a reduction in design inertia \( k_h \) used in conjunction with the M-O method, to \( 0.5 \times \text{PGA} \), implicitly allowing for small co-seismic displacements to occur during the design earthquake event (typically less than 10cm and therefore negligible for most applications). A simple means to estimate the order of magnitude of the displacements is also provided. However, the meaning of this reduction and the estimate of displacement is ambiguous for embedded walls, and caution is advised. It is recommended that the mode of deformation be considered, and for embedded walls the method of Veletsos & Younan [18] based on a shear beam model may be more appropriate.

If one wishes to consider a Newmark analysis directly, or using one of the many published empirical methods to assess sliding displacements, the implicit reduction factor in EC8 should be removed, and partial factors set to unity, prior to calculation of the critical acceleration. For tilting mode displacements, the method of Steedman and Zeng [44] (see also [45]) may be applied where the wall is situated on a rigid founding stratum. For combined tilting and bearing mode displacements (most applications) a modified method may be developed based on the same concepts. An alternative to the above simple rigid block models is to use a fully dynamic numerical analysis, the details of which is beyond the scope of this paper, but there are many examples in the literature.

**Conclusions**

This technical note summarises the common methods to assess dynamic earth pressures and their limitations. It also provides reference to modifications and enhancements to the base design methods in order to account for these limitations. Where possible, reference is made to Eurocode 8 to note to what extent these developments have been incorporated into modern design. Hopefully this paper provides a useful reference for understanding the subtleties of dynamic earth pressure evaluation and updates the reader on recent developments.
References


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