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Importance of material properties in fabric structure design & analysis

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Abstract

Coated woven fabrics are used in state-of-the-art structures yet broad assumptions are made in both material testing and analysis. Design is not codified and relies heavily on experience and precedent. Increasingly architects are moving away from conventional fabric forms, often utilising lower levels of curvature and new materials. The result is less efficient, highly stressed structures which may be more sensitive to (poorly quantified) fabric material properties. This paper considers the importance of material properties and structural geometry in the design and analysis of tensile fabric structures. Three typical tensile forms (conic, hypar & barrel vault) have been considered. Recommendations are given on the types of structure that are sensitive to variability in material properties, and ‘rules of thumb’ are proposed for the efficient design of fabric structures.

Keywords: fabric structure, tensile structure, efficient design, hypar, conic, barrel vault.

1. Introduction

1.1. Architectural fabric material behaviour

Coated woven fabrics are used in state-of-the-art structures yet broad assumptions are made in both material testing and analysis. A combination of non-linear stress-strain response of the component materials (yarn and coating), combined with the interaction of orthogonal yarns, results in complex (non-linear, hysteretic, anisotropic) material behaviour (Bridgens, Gosling et al. 2004 [2]). Full quantification of the response of coated woven fabrics to in-plane loading (biaxial and shear) is time consuming and costly, and arguably has not yet been achieved. Even if comprehensive test data were available, techniques to utilize this data in structural analysis are in their infancy.

1.2. Tensile structural forms

Architectural fabrics have negligible bending or compression stiffness. The shape of the fabric canopy is therefore fundamental to its ability to resist all applied loads in tension. To resist both uplift and down-forces the surface of the canopy must be double-cu prestressed (Bridgens, Gosling et al. 2004 [3]). Boundary conditions determine t shape and stress distribution; ideally a uniform prestress is applied to the fabric. To a uniform prestress the fabric must take the form of a minimal surface. Early tensile structures used soap bubbles to determine this form, in a process known finding (Otto 1967 [6]). The minimal surface joins the boundary points with the possible membrane area and has uniform in-plane tensile stresses throughou fundamental forms of fabric structure can be developed by manipulating the conditions of a flat panel (Figure 1).

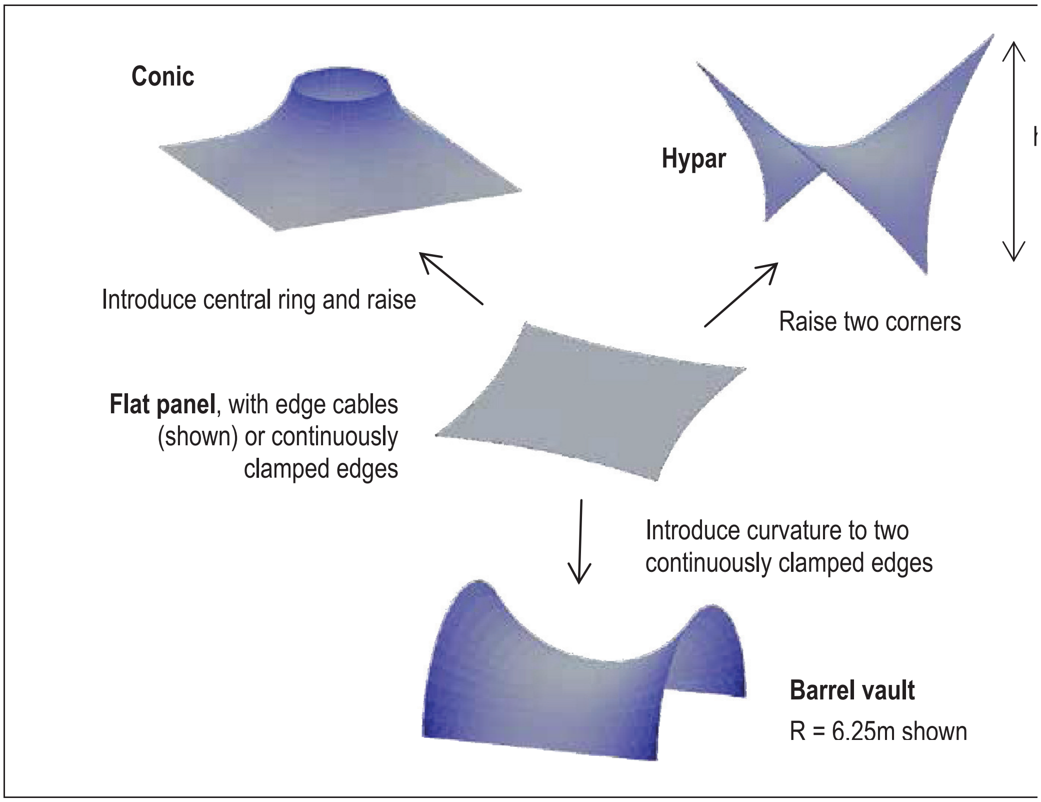

Figure 1: Development of tensile fabric forms

2. Scope & methodology

2.1. Material properties

A wide range of material properties have been used to investigate the effect variations which have been observed at different stress ratios and magnitudes (Gos

2.2. Form

To reduce the number of parameters being investigated all structures are square on plan. A stress ratio of 1:1 has been used throughout due to the difficulty of form finding at other prestress ratios using GSA (so-called pseudo soap-film form-finding). Figure 2 provides a summary of the geometric parameters used for each structural form.

Structure type	**Parameters (variable)**	**Parameters (constant)**	
Conic	Ring height (zero to maximum feasible - §3.3), ring diameter (zero to 14m)	Base: fixed edges, 14m x 14m Ring: fixed (not suspended)	Base: size, aspect ratio (square) Fabric prestress ratio & magnitude (3 kN/m in warp and fill)
Hypar	Corner height (h, Figure 1, zero to 8m) Patterning direction (diagonal or orthogonal, Figure 5)	Cable supported edges (constant cable prestress) Symmetrical structures, two diagonally opposite high points Base: 7.07m x 7.07m	
Barrel vault	Fabric radius of curvature, controlled by varying the arch radius of curvature. $R_{warp} = R_{fill}$. Zero (flat panel) to 6.25m (Figure 1)	Base: fixed edges, 10m x 10m	

Figure 2: Geometric model parameters

2.3. Loading

Critical loadcases for fabric structures are usually wind and snow loading. Even if wind-structure interaction and dynamic effects are ignored, accurate determination of wind loading for fabric structure forms is difficult (Burton and Gosling 2004 [4]) and there is limited design guidance. For this work a simplified approach has been adopted: all structures have been analysed for uniform wind uplift (1.0 kN/m^2) and uniform snow load (0.6 kN/m^2). Wind load is a suction force which acts perpendicular to the fabric surface, and has been applied using deformed, local coordinates, i.e. the wind load direction will be updated during the analysis as the structure deforms. This is consistent with the geometrically non-linear analysis (§2.4).

2.4. Modelling of tensile fabric structures

Modelling and analysis has been carried out using *Oasys GSA* software (www.oasys.com/gsa). The first stage is to define the boundary conditions (geometry, fixed or cable edges) & form finding properties (fabric and edge cable prestress forces). A soap soap-film form-finding analysis (Barnes 1999 [1]) provides the fabric geometry and prestress loads. Finally the fabric material properties are defined, loads are applied (wind, snow, prestress) and a geometrically non-linear (large displacement) analysis is carried out.

3. Results & discussion

3.1. Edge cable curvature

Edge cable curvature and cable tension are related by equation (1) which defines a linear variation of tension with curvature. However, for architectural design it is the 'dip' (Figure 3) that is significant, as this determines the level of coverage and aesthetics of the canopy.

$$\textit{Tension} = \textit{uniform applied load} \times \textit{radius of curvature} \tag{1}$$

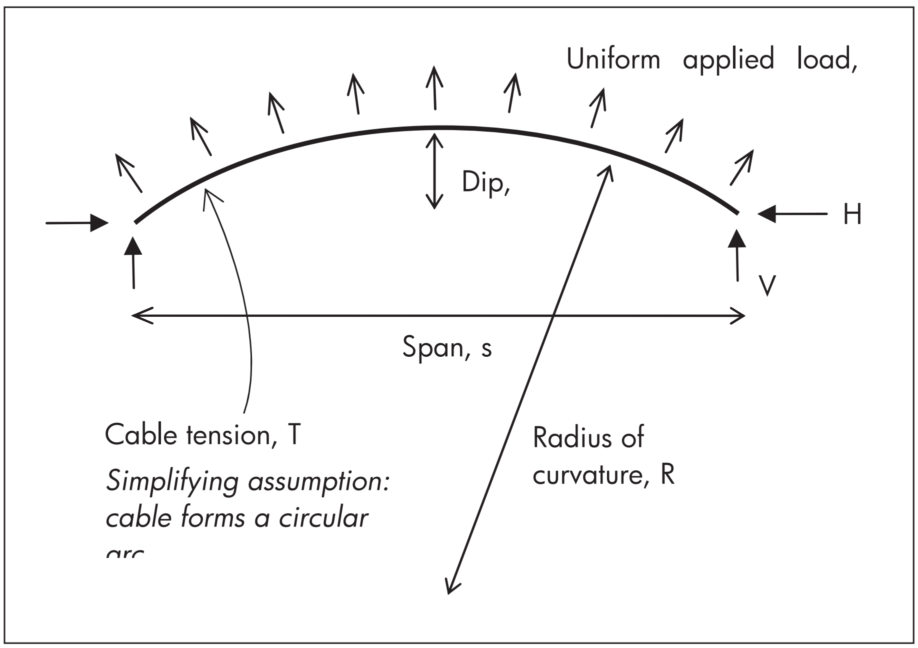

Figure 3: Edge cable curvature

Using the simplifying assumption that the edge cable forms a circular arc, the cable tension, T, can be written in terms of the end reactions (H &V):

$$T = \sqrt{H^2 + V^2}, \quad H = \frac{ws^2}{8d}, \quad V = \frac{ws}{2} \tag{2}$$

Refer to Figure 3 for nomenclature. Substituting gives an expression for T in terms of dip (d), span (s) and applied load (w), and the resulting relationship between cable force and dip is shown in Figure 4.

$$T = \sqrt{\left(\frac{ws^2}{8d}\right)^2 + \left(\frac{ws}{2}\right)^2} \tag{3}$$

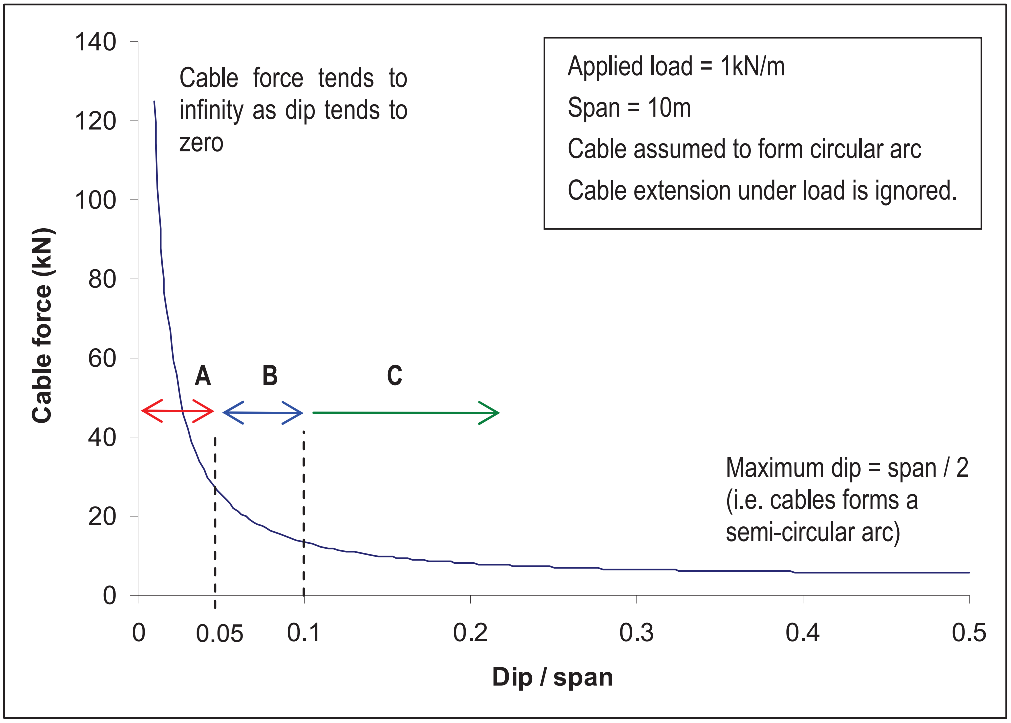

Figure 4: Variation of cable force with dip

A dip to span ratio greater than 0.1 (‘C’, Figure 4) ensures low cable force and hence an efficient transfer of fabric stress back to the supporting structure. This will result in smaller diameter cables, smaller end fittings, and consequently smaller, more elegant connection details and supporting steelwork. A dip to span ratio of 0.5 to 0.1 (‘B’) may be desirable for architectural reasons, for example to provide good coverage, but it should be noted that over this range the cable force doubles. A dip/span ratio less than 0.5 (‘A’) should be avoided as the cable force increases dramatically.

3.2. Hypar

3.2.1. Patterning

A hypar can be designed with two different fabric orientations or patterning directions (Figure 5). A square hypar structure acts principally in tension between diagonally opposite corners. For the orthogonally patterned hypar (Figure 5a) this means that the fabric is acting in shear. As the shear stiffness is typically low (elastic modulus ÷ 20 is commonly used as a rule of thumb) an orthogonally patterned hypar will exhibit high deflections (Figure 6).

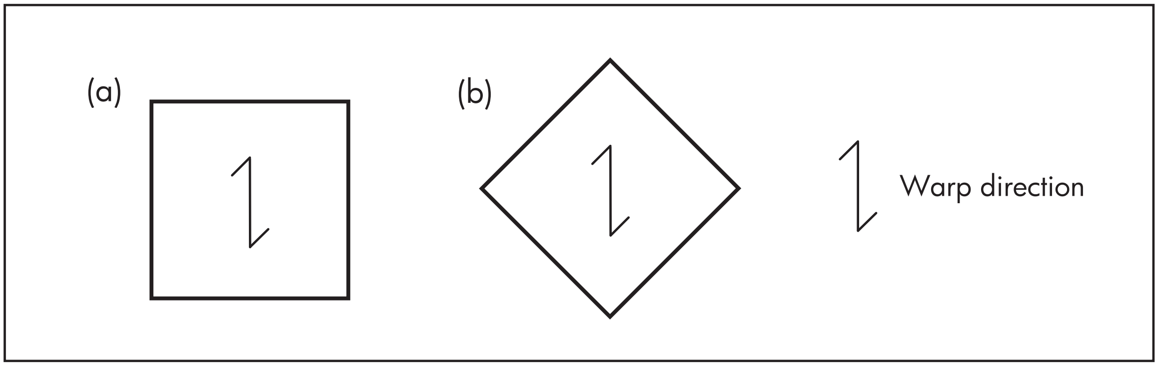

Figure 5: Hypar patterning options, (a) orthogonal, and (b) diagonal

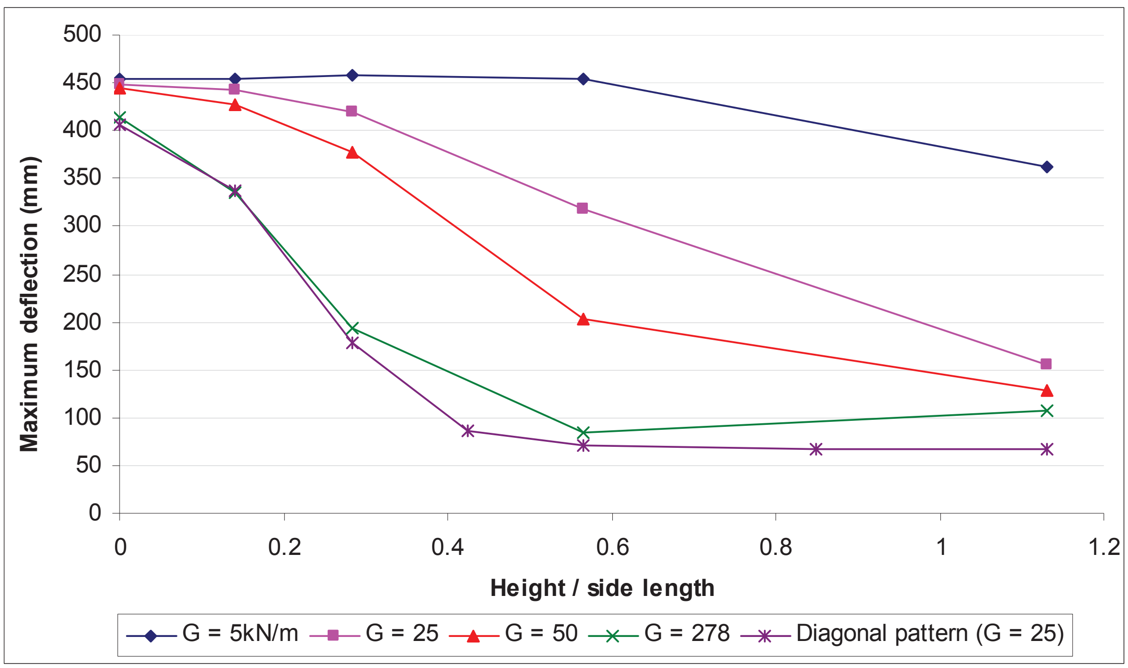

Figure 6: Square hypar, orthogonal patterning, variation of deflection due to wind uplift with height and shear modulus

As the shear modulus increases towards the value for an isotropic material, equation (4), the shear stiffness tends towards the elastic stiffness in warp and fill directions, and hence the displacements tend towards the values for the diagonally patterned structure (Figure 6).

$$G = \frac{E}{2(1+\upsilon)} \tag{4}$$

Where G = shear modulus, E = elastic modulus, υ = Poisson's ratio.

This effect is less significant at low heights (i.e. as height / side lengths tends to zero) when the fabric panel will be acting primarily as a two way spanning flat panel. As the corner height and fabric curvature increases (right hand side of Figure 6), the structure must span between diagonally opposite corners and the effect of fabric orientation and shear stiffness becomes pronounced.

3.2.2. Significance of corner height and material properties

The behaviour of a hypar varies considerably from a flat or near-flat panel which spans in two directions, to a true hypar which resists load as tension between two opposite corners (Figure 7 to Figure 9). A wide variation in fabric stiffness has been modelled, from 100 kN/m through realistic values of 400 kN/m to 2000 kN/m, to a maximum of 5000 kN/m. For hypars with a high level of fabric curvature the sensitivity to changes in elastic modulus are very low. However, as the curvature is reduced and the behaviour tends towards that of a flat panel, the elastic modulus becomes much more significant.

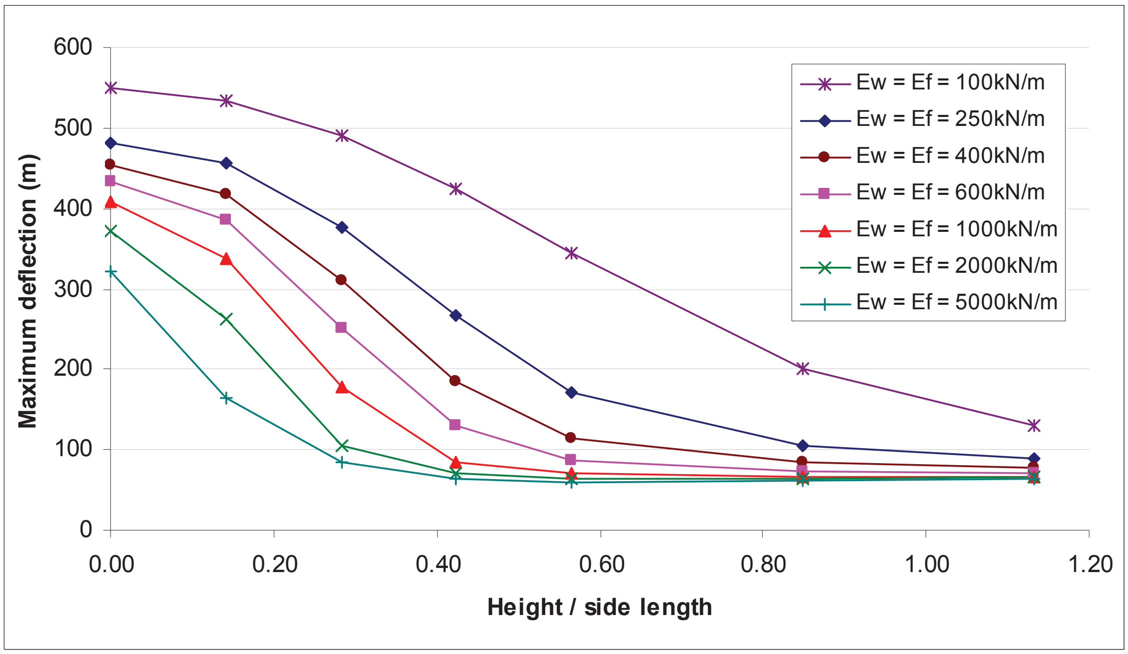

Figure 7: Square hypar, diagonal patterning, variation of deflection due to wind uplift with height and elastic modulus

Hypars have been analysed with a wide variation in fabric shear modulus (from 5 kN/m, through realistic values of 25 kN/m to 50 kN/m, to a maximum of the isotropic value of 278 kN/m). The effect on deflections is small (results not shown) but the effect on fabric stress is more significant (Figure 10, results shown for fill direction only), which is concerning in the context that shear stiffness is rarely tested for architectural fabrics and assumed values are used in analysis.

Poisson's ratio was varied (from 0.1 to 0.9) but was found to not have a significant effect on stress or deflection levels.

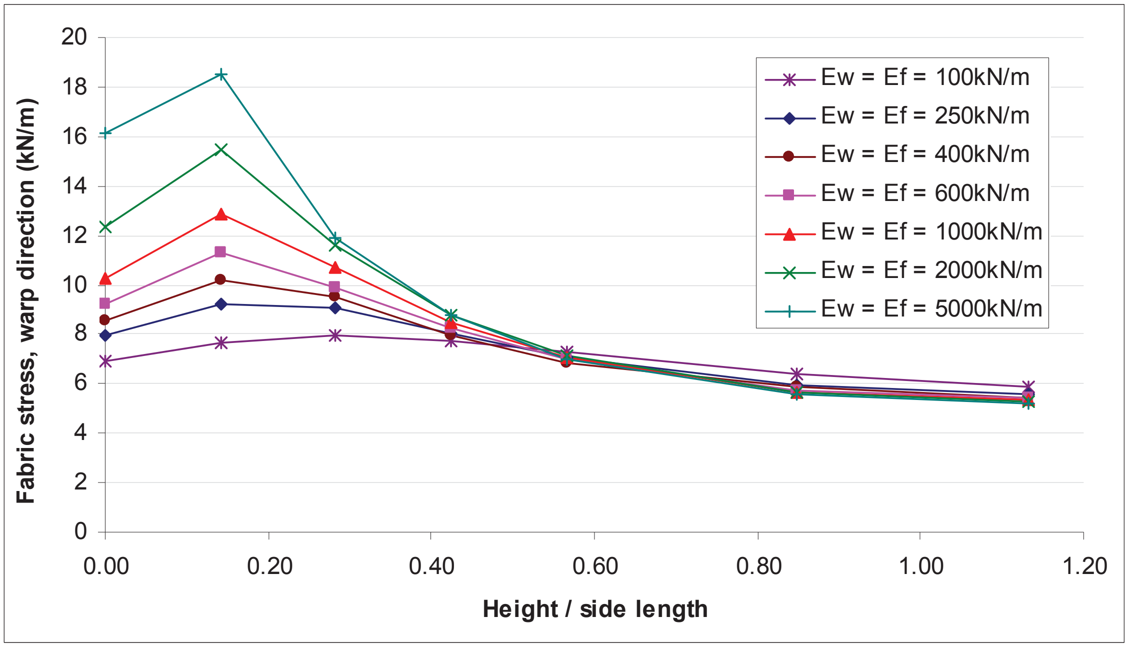

Figure 8: Square hypar, diagonal patterning, variation of fabric stress (warp direction) due to wind uplift with height and elastic modulus

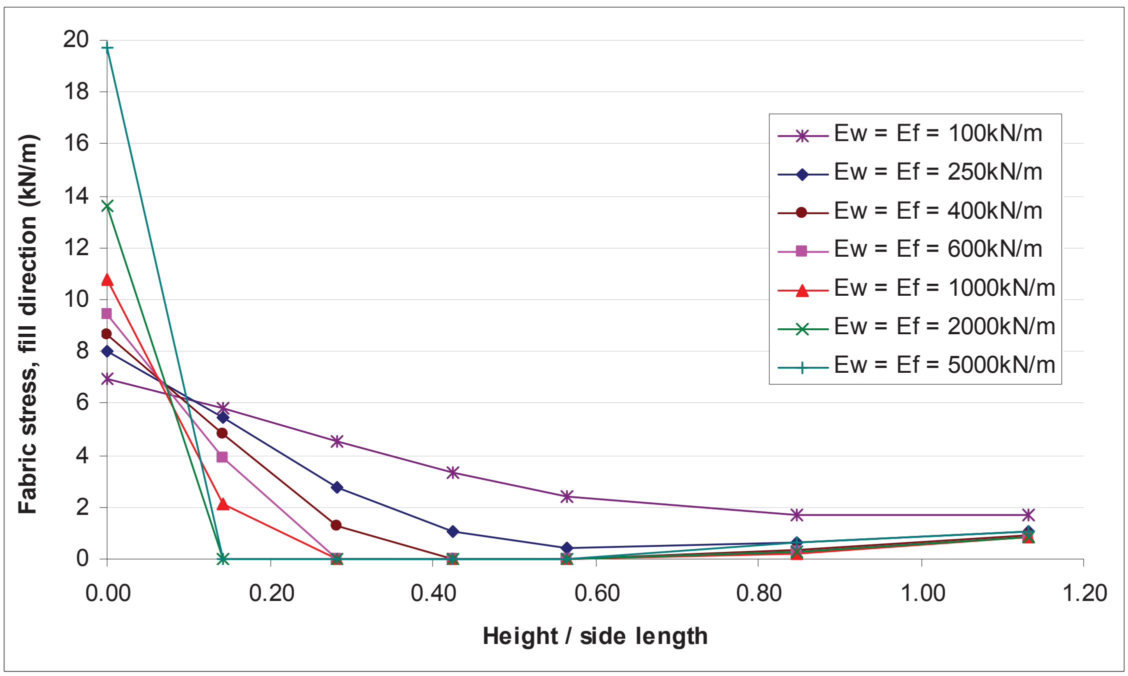

Figure 9: Square hypar, diagonal patterning, variation of fabric stress (fill direction) due to wind uplift with height and elastic modulus

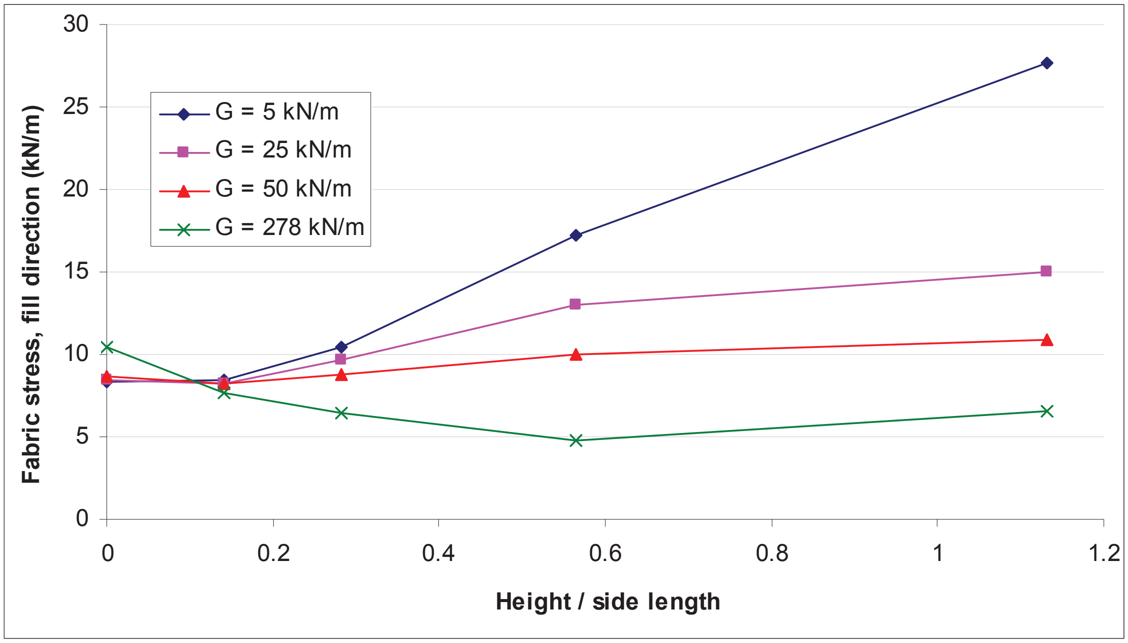

Figure 10: Square hypar, orthogonal patterning, variation of fabric stress (fill direction) due to wind uplift with height and shear modulus

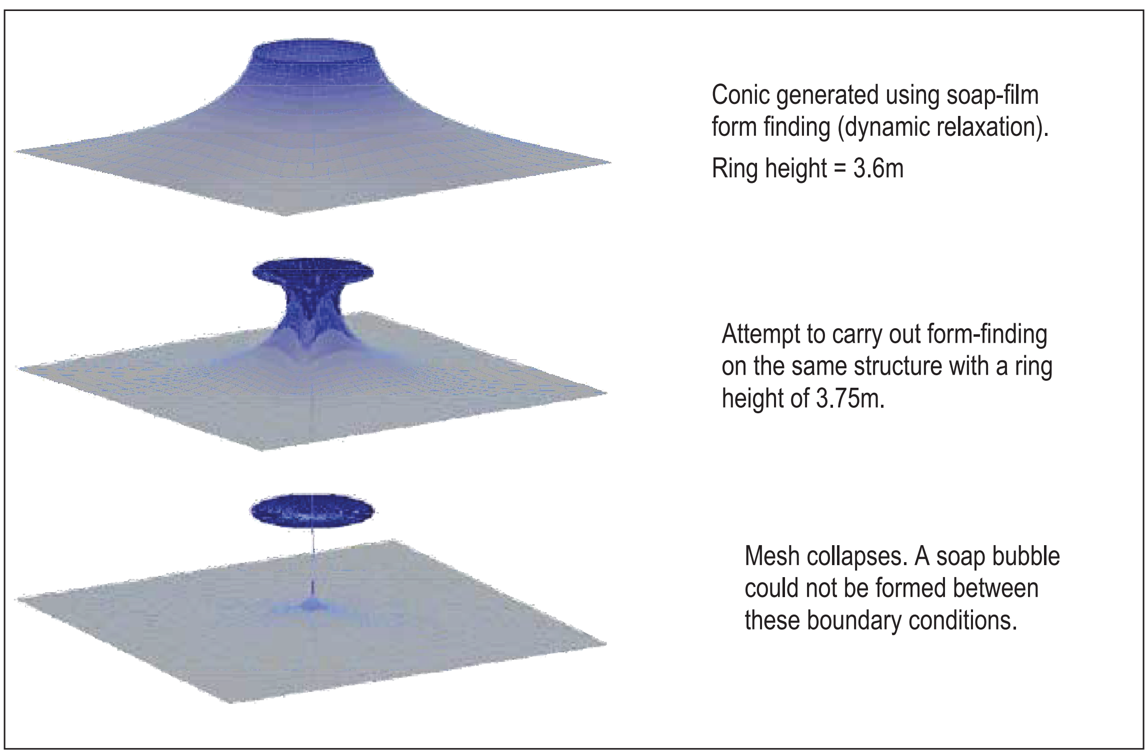

Figure 11: Limitations of soap-film surfaces

3.3. Conic

A true minimal surface cannot be formed between all boundary conditions. As the distance between the base and top ring increases the minimal surface will 'neck': a point is reached where a minimal surface cannot be formed between the rings (Figure 11). A pseudo-minimal surface can be developed for a fabric membrane by accepting increased stresses in the region where the soap bubble would have failed, reducing the limitations on the forms that can be created. However, as the desired shape moves away from the minimal surface, the stress variations increase and the structure becomes less efficient. The feasible bounds of conic geometry have been determined for a prestress ratio of 1:1 (Figure 12). Geometric properties are given as ratios of ring diameter and height to base edge length because the results are generally applicable to any scale of structure.

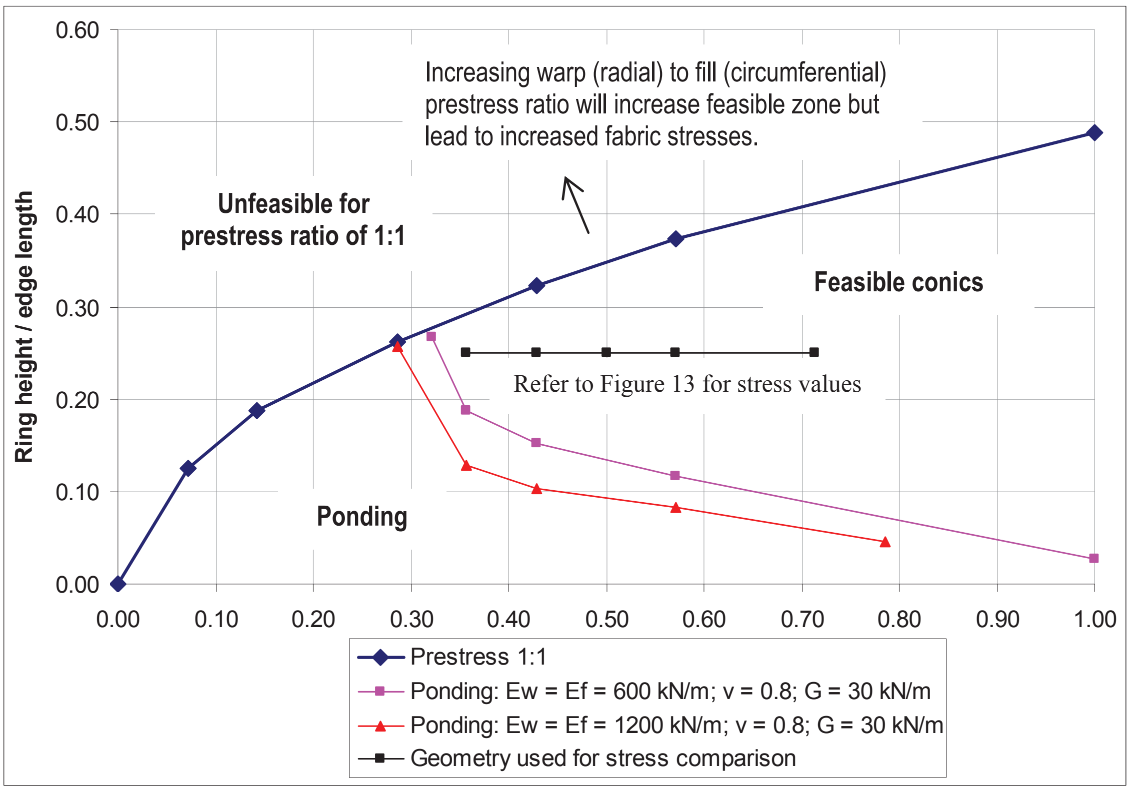

Figure 12: Limiting values of conic geometry for form finding (1:1 prestress) and ponding

Conic structures with a low ring are prone to ponding - formation of a hollow near the corners under snow load which leads to collection of melt-water and subsequent failure. Ponding checks have been used to further refine the feasible conic geometries for typical fabric properties that are used for analysis of PVC coated polyester and PTFE coated glass-fibre structures (Figure 12). Within the 'feasible zone' the variation of fabric stress with geometry has been assessed (Figure 13).

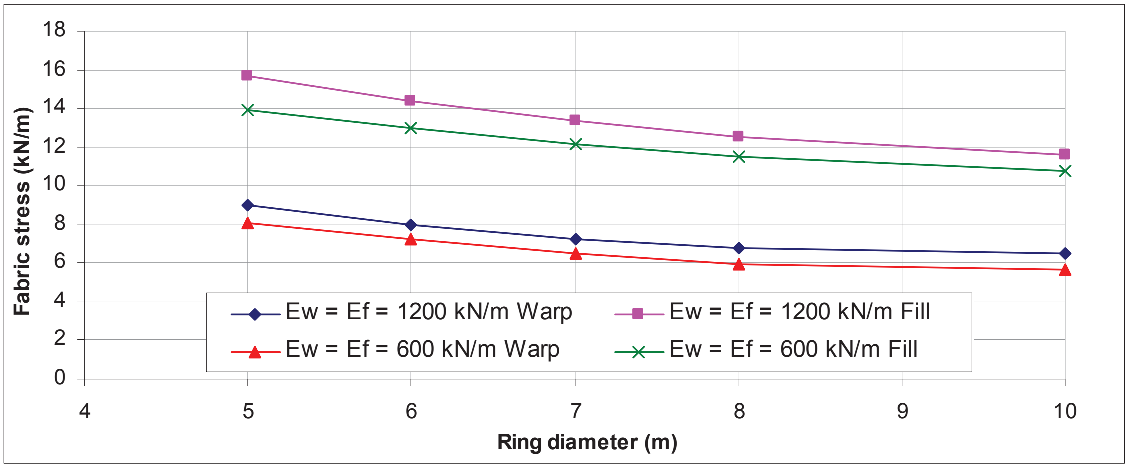

Figure 13: Variation of stress with ring diameter & elastic modulus; 3.5m ring height.

3.4. Barrel vault

The barrel vault has been analysed with a combination of elastic moduli values and fabric curvatures. A small fabric radius (i.e. highly curved) provides an efficient structure with low values of stress and deflection (Figure 14). As the barrel vault flattens and tends towards a flat panel, the fabric stresses and deflections increase by a factor of between 2 and 3. At the same time, the effect of fabric stiffness becomes much more significant as the curvature is reduced.

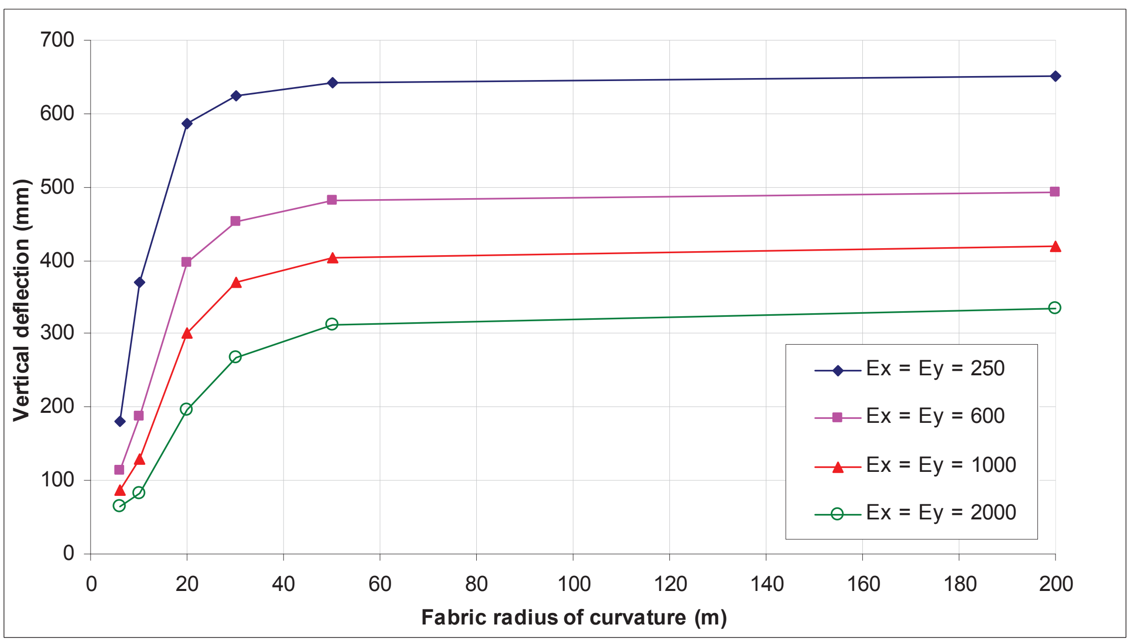

Figure 14: Barrel vault, variation of deflection with fabric curvature & elastic modulus

4. Conclusions

The overall conclusion for all structural forms is that as fabric curvature decreases stress levels and deflections increase, and the sensitivity to variations in material properties increases. More detailed conclusions are summarised below:

- For efficient edge cables the value of dip/span should always be greater than 0.05 and preferably greater than 0.1,
- Hypar patterning: orthogonal patterning works for very low hypars which act as a flat panel, but diagonal patterning (i.e. warp and fill run between diagonally opposite corners) should be used for all other hypars to minimise deflections,
- Values of elastic modulus have a dramatic effect on hypar deflections up to a height/side length ratio of 0.4,
- Shear modulus has a significant effect on hypar stresses, in particular at high curvatures (height/side length > 0.5),
- Feasible conic forms with a prestress ratio of 1:1 are severely limited by form finding and ponding constraints. Work is ongoing to determine the effect of variations in prestress ratio on feasible forms and stress levels,
- The stress and deflection levels in a barrel vault increase by a factor of between two and three as the curvature reduces, and the sensitivity to changes in fabric stiffness increases.

5. References

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