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Concrete Code Reference

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Contents

Notation

Symbol	Represents
f_c	Concrete strength (with code variations)
f	Concrete Stress
ε	Concrete strain
ε_c	Strain at which concrete stress is maximum
ε_{cu}	Strain at which concrete fails
ρ	Density
t	Time
φ	Creep factor

Design Codes

The following design codes are supported to varying degrees in the Oasys software products.

Code	Title	Country	Date	Other versions
ACI318	Building Code Requirements for Structural Concrete (ACI318-14)	USA	2014	2011, 2008, 2005, 2002
ACI318M	Building Code Requirements for Structural Concrete (ACI318M-14) (metric version)	USA	2014	2011, 2008, 2005, 2002
AS3600	Australian Standard Concrete Structures 2009	Australia	2009	2001
BS5400-4	Steel, concrete and composite bridges – Code of practice for design of concrete bridges	UK	1990	IAN70/06
BS8110-1	Structural Use of Concrete Part 1. Code of practice for design and construction (Incorporating Amendments Nos. 1, 2 and 3)	UK	2005	1997, 1985
BS EN 1992-1-1	Eurocode 2-1	UK	2004	PD6687:2006
BS EN 1992-2	Eurocode 2-2	UK	2005	
CAN CSA A23.3	Design of Concrete Structures	Canada	2014	2004
CAN CSA S6	Canadian Highway Bridge Design Code	Canada	2014	
CYS EN 1992-1-1	Eurocode 2-1-1	Cyprus	2004	
DIN EN 1992-1-1	Eurocode 2-1-1	Germany	2004	
DIN EN 1992-2	Eurocode 2-2	Germany	2010	
DS/EN 1992-1-1	Eurocode 2-1-1	Denmark	2004	
DS/EN 1992-2	Eurocode 2-2	Denmark	2005	
EN1992-1-1	Eurocode 2: Design of concrete structures – Part 1-1: General rules and rules for buildings		2004	
EN 1992-2	Eurocode 2: Design of concrete structures. Concrete bridges - Design and detailing rules		2005	
Hong Kong Code of Practice	Code of Practice for the Structural Use of Concrete	Hong Kong	2013	2004 (AMD 2007), 2004, 1987

Hong Kong Structures Design Manual	Structures Design Manual for Highways and Railways	Hong Kong	2013	2002
IRC:112	Code of Practice for Concrete Road Bridges	India	2011	
IRS Concrete Bridge Code	Code of Practice for Plain, Reinforced & Prestressed Concrete for General Bridge Construction	India	1997	
I.S. EN 1992-1-1	Eurocode 2-1-1	Ireland	2004	
I.S. EN 1992-2	Eurocode 2-2	Ireland	2005	
IS 456	Plain and reinforced concrete – Code of Practice	India	2000	
NEN EN 1992-1-1	Eurocode 2-1-1	Netherlands	2004	
NEN EN 1992-2	Eurocode 2-2	Netherlands	2011	
NF EN 1992-1-1	Eurocode 2-1-1	France	2005	
NF EN 1992-2	Eurocode 2-2	France	2006	
NS-EN 1992-1-1	Eurocode 2-1-1	Norway	2004	
PN-EN 1992-1-1	Eurocode 2-1-1	Poland	2008	
SFS-EN1992-1-1	Eurocode 2-1-1	Finland		
UNE-EN 1992-1-1	Eurocode 2-1-1	Spain	2010/ 2013	
UNE-EN 1992-2	Eurocode 2-2	Spain	2010	
UNI EN 1992-1-1	Eurocode 2-1-1	Italy	2004	
UNI EN 1992-2	Eurocode 2-2	Italy	2006	

Concrete material models

Units

The default units are:

Stress, strength   MPa (psi)

Elastic modulus   GPa (psi)

Concrete material models for different codes

Different material models are available for different design codes. These are summarised below:

	ACI 318	AS 3600	BS 5400	BS 8110	CSA A23.3	CSA S6	EN 1992	HK CP	HK SDM	IRC:112	IRS Bridge	IS 456
Compression												
Parabola-rectangle	●	●	●	●	●	●	●	●	●	●	●	●
Rectangle	●	●		●	●	●	●	●		●		●
Bilinear							●			●		
Linear	●	●	●	●	●	●	●	●	●	●	●	●
FIB				●			●	●		●		●
Popovics	●	●			●	●						
EC2 Confined							●			●		
AISC 360 filled tube	●											
Explicit	●	●	●	●	●	●	●	●	●	●	●	●
Tension												
No-tension	●	●	●	●	●	●	●	●	●	●	●	●
Linear	●	●		●	●	●	●	●		●		●
Interpolated	●	●			●	●	●			●		
BS8110 - 2				●				●				●

TR 59			●				●				●
PD 6687						●					
Explicit	●	●	●	●	●	●	●	●	●	●	●	●
Explicit envelope	●	●	●	●	●	●	●	●	●	●	●	●

inferred from rectangular block

PD 6687 variant of EN 1992 only

Parabola-rectangle

Parabola-rectangles are commonly uses for concrete stress-strain curves.

Parabola-rectangle
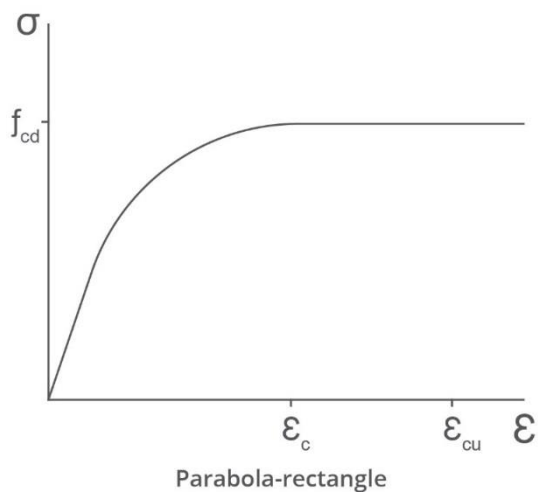
The parabolic curve can be characterised as

$$\frac{f}{f_{cd}} = a\left(\frac{\varepsilon}{\varepsilon_c}\right)^2 + b\left(\frac{\varepsilon}{\varepsilon_c}\right)$$

At strains above ε_c the stress remains constant. For most design codes the parabola is taken as having zero slope where it meets the horizontal portion of the stress-strain curve.

$$\frac{f}{f_{cd}} = \left[1 - \left(1 - \frac{\varepsilon}{\varepsilon_c}\right)^2\right]$$

The Hong Kong Code of Practice (supported by the Hong Kong Institution of Engineers) interpret the curve so that the initial slope is the elastic modulus (meaning that the parabola is not tangent to the horizontal portion of the curve).

$$\frac{f}{f_{cd}} = \left[1 - \left(\frac{E}{E_s}\right)\right]\left(\frac{\varepsilon}{\varepsilon_c}\right)^2 + \left(\frac{E}{E_s}\right)\left(\frac{\varepsilon}{\varepsilon_c}\right)$$

where the secant modulus is

$$E_s = \frac{f_{cd}}{\varepsilon_c}$$

In Eurocode the parabola is modified

$$\frac{f}{f_{cd}} = \left[1 - \left(1 - \frac{\varepsilon}{\varepsilon_c}\right)^n\right]$$

and

$$n = 2 \qquad\qquad f_c \leq 50\text{MPa}$$

$$n = 1.4 + 23.4\left[(90 - f_c)/100\right]^4 \qquad f_c > 50\text{MPa}$$

EC2 Confined

The EC2 confined model is a variant on the parabola-rectangle. In this case the confining stress σ increases the compressive strength and the plateau and failure strains.

$$f_{c,c} = \begin{cases} f_c(1 + 5\sigma/f_c) & \sigma \leq 0.05\,f_c \\ f_c(1.125 + 2.5\sigma/f_c) & \sigma > 0.05\,f_c \end{cases}$$

$$\varepsilon_{c,c} = \varepsilon_c\left(f_{c,c}/f_c\right)^2$$

$$\varepsilon_{cu,c} = \varepsilon_{cu} + 0.2\,\sigma/f_c$$

Rectangle

The rectangular stress block has zero stress up to a strain of ε_c (controlled by β) and then a constant stress of $\alpha\,f_{cd}$.

Rectangular
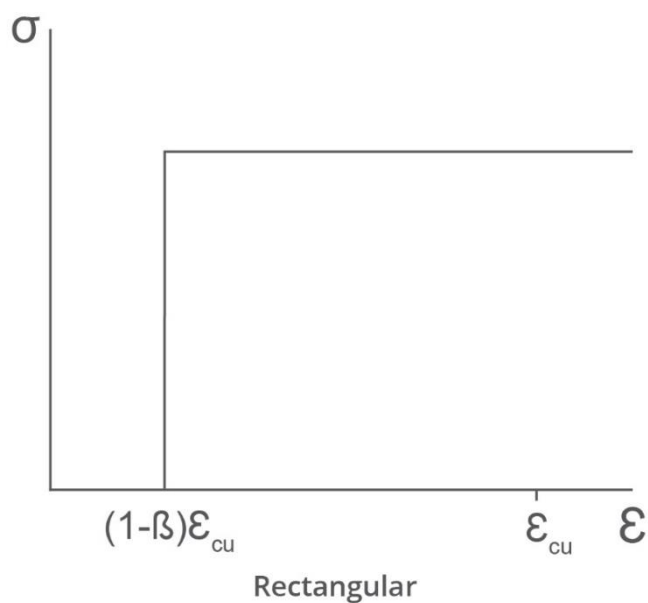
		α	β
ACI 318		1	0.85 - 0.05(f_c - 30)/7 [0.65:0.85]
AS3600	2001	1	0.85 - 0.07(f_c - 28) [0.65:0.85]
AS3600	2009	1	1.05 - 0.007f_c [0.67:0.85]
BS5400		0.6/0.67	1
BS8110		1	0.9
CSA A23.3		1	max(0.67, 0.97 - 0.0025 f_c)
CSA S6		1	max(0.67, 0.97 - 0.0025 f_c)
EN 1992		1 $f_c \leq$ 50MPa 1 - (f_c - 50)/200 f_c> 50MPa	0.8 $f_c \leq$ 50MPa 0.8 - (f_c - 50)/400 f_c > 50MPa
HK CP	> 2004	1	0.9
HK CP	2007 >	1	0.9 $f_c \leq$ 45MPa 0.8 $f_c \leq$ 70MPa 0.72 $f_c \leq$ 100MPa

HK SDM		0.6/0.67		1	
IRC:112		1 $\quad\quad\quad\quad$ $f_c \leq$ 60MPa 1 - (f_c - 60)/250 $\quad$ $f_c >$ 60MPa		0.8 $\quad\quad\quad\quad$ $f_c \leq$ 60MPa 0.8 - (f_c - 60)/500 $\quad\quad$ $f_c >$ 60MPa	
IRS Bridge		0.6/0.67		1	
IS 456		0.8		0.84	

Bilinear

The bilinear curve is linear to the point (ε_c, f_{cd}) and then constant to failure.

Bilinear
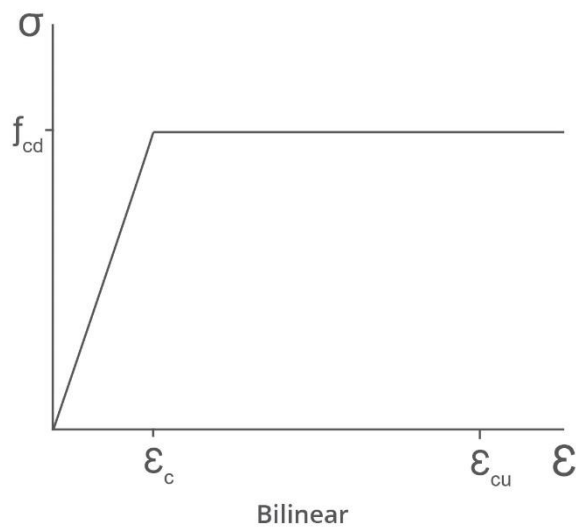
FIB

The FIB model code defines a schematic stress-strain curve. This is used in BS 8110-2, EN1992-1 and IRC:112.

FIB Schematic
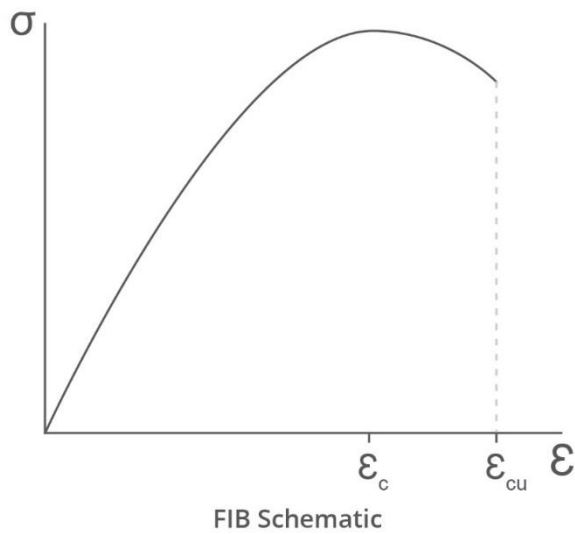
This has a peak stress f_{cFIB}

This is defined as

$$\frac{f}{f_{cFIB}} = \frac{k\eta - \eta^2}{1 + (k-2)\eta}$$

with

$$k = \alpha \frac{E_c}{f_c / \varepsilon_c}$$

Where the factor α is code dependent.

Code	f_{cFIB}	α
BS 8110-2	$0.8 f_c$	1.4
EN 1992-1	$f_c + 8\,MPa$	1.05
IRC:112	$f_c + 10\,MPa$	1.05

Popovics

There are a series of curves based on the work of Popovics.

Popovics
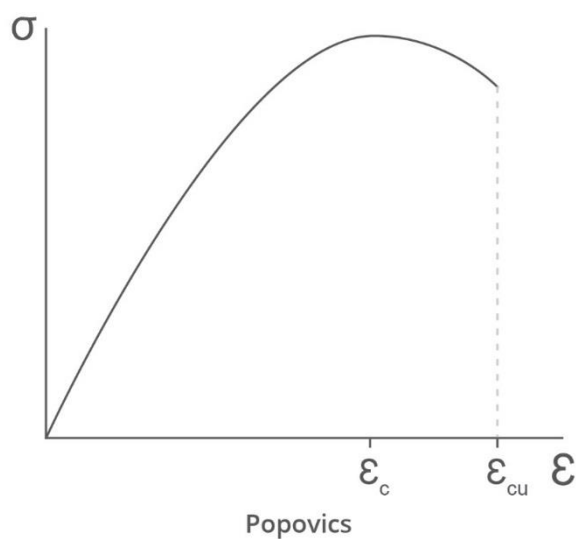
These have been adjusted and are based on the Thorenfeldt base curve.

In the Canadian offshore code (CAN/CSA S474-04) this is characterised by

$$\frac{f}{f_c} = k_3 \eta \, \frac{n}{n - 1 + \eta^{nk}}$$

with (in MPa)

$$\eta = \frac{\varepsilon}{\varepsilon_c}$$

$$k_3 = 0.6 + \frac{10}{f_c}$$

$$n = 0.8 + \frac{f_c}{17}$$

$$\varepsilon_c = \frac{f_c}{E_c} \frac{n}{n - 1}$$

$$k = \begin{cases} 1 & \eta \leq 1 \\ 0.67 + \dfrac{f_c}{62} & \eta > 1 \end{cases}$$

The peak strain is referred to elsewhere as ε_{pop} .

$$\varepsilon_{pop} = \varepsilon_c$$

All the concrete models require a strength value and a pair of strains: the strain at peak stress or transition strain and the failure strain.

Mander & Mander confined curve

The Mander[1] curve is available for both strength and serviceability analysis and the Mander confined curve for strength analysis.

Mander
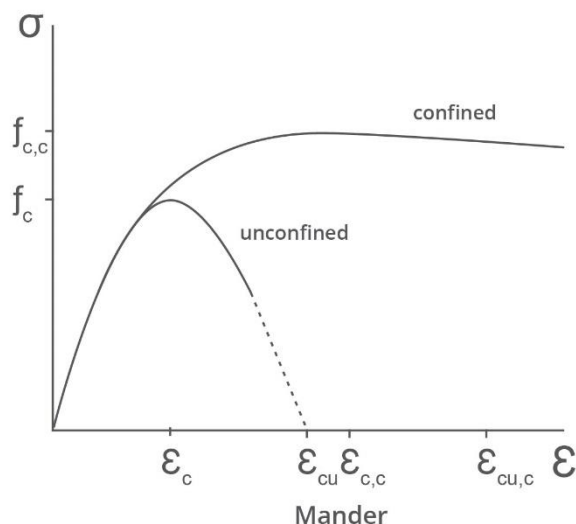
For unconfined concrete, the peak of the stress-strain curve occurs at a stress equal to the

unconfined cylinder strength f_c and strain ε_c generally taken to be 0.002. Curve constants are calculated from

$$E_{sec} = f_c / \varepsilon_c$$

and

$$r = \frac{E}{E - E_{sec}}$$

Then for strains $0 \leq \varepsilon \leq 2\varepsilon_c$ the stress σ can be calculated from:

$$\sigma = f_c \frac{\eta\, r}{r - 1 + \eta^{\,r}}$$

where

$$\eta = \frac{\varepsilon}{\varepsilon_c}$$

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[1] Mander J, Priestly M, and Park R. Theoretical stress-strain model for confined concrete. Journal of Structural Engineering, 114(8), pp1804-1826, 1988.

The curve falls linearly from $2\varepsilon_c$ 2eco to the 'spalling' strain ε_{cu}. The spalling strain can be taken as 0.005-0.006.

To generate the confined curve the confined strength $f_{c,c}$ must first be calculated. This will depend on the level of confinement that can be achieved by the reinforcement. The maximum strain $\varepsilon_{cu,c}$ also needs to be estimated. This is an iterative calculation, limited by hoop rupture, with possible values ranging from 0.01 to 0.06. An estimate of the strain could be made from EC2 formula (3.27) above with an upper limit of 0.01.

The peak strain for the confined curve $\varepsilon_{c,c}$ is given by:

$$\varepsilon_{c,c} = \varepsilon_c \left[1 + 5\left(\frac{f_{c,c}}{f_c} - 1 \right) \right]$$

Curve constants are calculated from

$$E_{sec} = f_{c,c} / \varepsilon_{c,c}$$

and

$$r = \frac{E}{E - E_{sec}}$$

as before.

E is the tangent modulus of the unconfined curve, given above.

Then for strains $0 \leq \varepsilon \leq \varepsilon_{cu,c}$ the stress σ can be calculated from:

$$\sigma = f_{c,c} \frac{\eta\, r}{r - 1 + \eta^r}$$

where

$$\eta = \frac{\varepsilon}{\varepsilon_{c,c}}$$

BS8110-2 tension curve

BS8110-2 define a tension curve for serviceability

BS8110-2
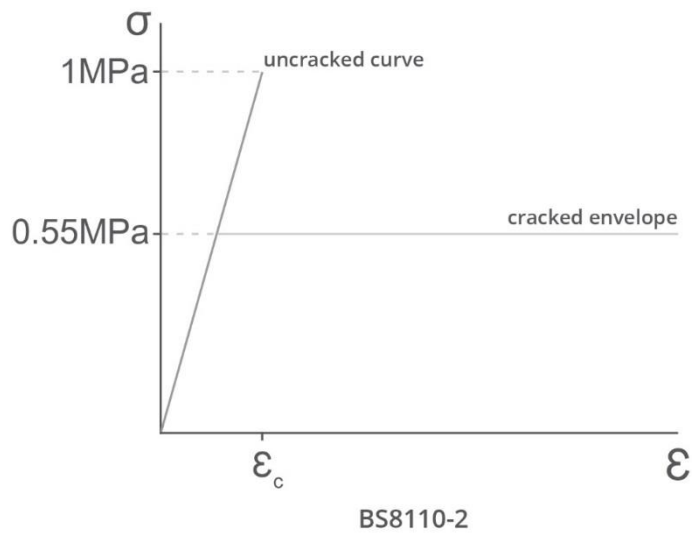
TR59

Technical report 59 defines an envelope for use with concrete in tension for serviceability. The material is assumed to behave in a linearly elastic manner, with the elastic modulus reduced beyond the peak stress/strain point based on the envelope in the figures below

Tension stiffening for short-term loads

Tension stiffening for long-term loads
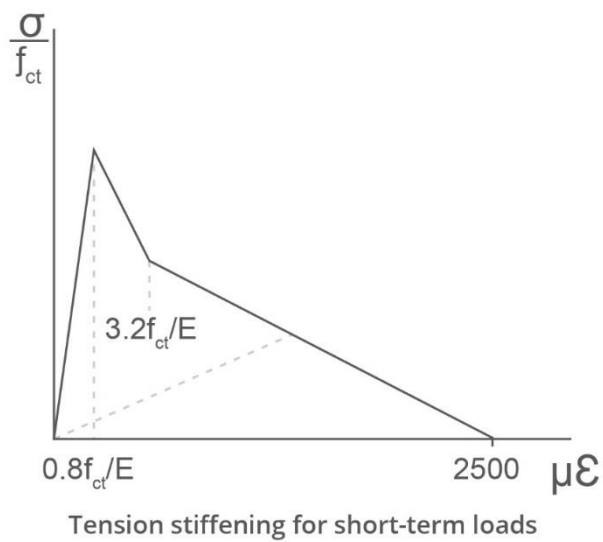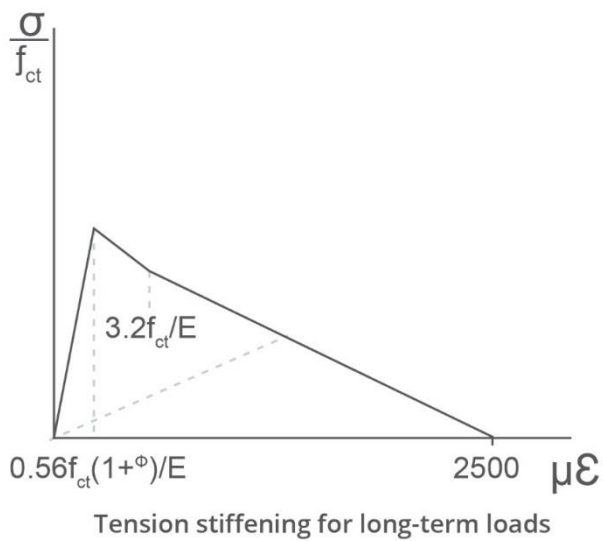
Interpolated

Interpolated strain plains to ACI318 and similar codes

ACI318 and several other codes give a method to compute a value of the second moment of area intermediate between that of the uncracked, I_g , and fully cracked, I_{cr} , values, using the following expression:

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right]I_{cr}$$

where M_{cr} is the cracking moment and M_a is the applied moment.

AdSec SLS analyses determine a strain plane intermediate to the uncracked and fully cracked strain planes. The program determines a value for ς , the proportion of the fully cracked strain plane to add to the proportion $(1 - \varsigma)$ of the uncracked plane so that the resulting plane is compatible with ACI318's approach. Unfortunately, since ACI318's expression is an interpolation of the inverse of the curvatures, rather than the curvatures themselves, there is no direct conversion. It should also be noted that although I_g is defined as the value of second moment of area ignoring the reinforcement, it is assumed that this definition was made for simplicity, and AdSec includes the reinforcement.

Let $\alpha = \left(M_{cr}/M_a\right)^3$, the uncracked curvature be κ_I and the fully cracked curvature be κ_{II} .

To ACI318, the interpolated curvature

$$\kappa = \frac{1}{\left[\alpha/\kappa_I + (1 - \alpha)/\kappa_{II}\right]},$$

and the aim is to make this equivalent to

$$\kappa = \zeta \kappa_{II} + (1 - \zeta)\kappa_I \,,$$

Equating these two expressions gives

$$\alpha\zeta \; \kappa_{II}/\kappa_I + \alpha(1 - \zeta) + (1 - \alpha)\zeta + (1 - \alpha)(1 - \zeta)\kappa_I/\kappa_{II} = 1$$

which can be re-arranged to give

$$\zeta = \frac{1}{[1 + (\kappa_{II}/\kappa_I)/(1/\alpha - 1)]}$$

The ratio κ_{II}/κ_I is appropriate for uniaxial bending. For applied loads (N, M_y, M_z), and uncracked and fully cracked strain planes $(\varepsilon_I, \kappa_{yI}, \kappa_{zI})$ and $(\varepsilon_{II}, \kappa_{yII}, \kappa_{zII})$ respectively, κ_{II}/κ_I is replaced by the ratio $(N\varepsilon_{II} + M_y\kappa_{yII} + + M_z\kappa_{zII})/(N\varepsilon_I + M_y\kappa_{yI} + + M_z\kappa_{zI})$, which is independent of the location chosen for the reference point. In the absence of axial loads, this ratio ensures that the curvature about the same axis as the applied moment will comply with ACI318; in the absence of moments, the axial strain will follow a relationship equivalent to that in ACI318 but using axial stiffness as imposed to flexural stiffness.

The ratio (M_{cr}/M_a) is also inappropriate for general loading. For the general case, it is replaced by the ratio (f_{ct}/σ_{tI}), where f_{ct} is the tensile strength of the concrete and σ_{tI} is the maximum concrete tensile stress on the uncracked section under applied loads.

Summary:
$$\zeta = 1 \bigg/ \left[1 + \frac{(N\varepsilon_{II} + M_y\kappa_{yII} + M_z\kappa_{zII})/(N\varepsilon_I + M_y\kappa_{yI} + M_z\kappa_{zI})}{(\sigma_{tI}/f_{ct})^3 - 1} \right]$$

Since ζ is larger for short-term loading, all curvatures and strains are calculated based on short-term properties regardless of whether ζ is subsequently used in a long-term serviceability calculation.

Concrete properties

Notation

f_c concrete strength

f_{cd} concrete design strength

f_{ct} concrete tensile strength

E elastic modulus

v Poisson's ratio (0.2)

α coefficient of thermal expansion (varies but 1×10^{-6}/°C assumed)

ε_{cu} strain at failure (ULS)

ε_{ax} compressive strain at failure (ULS)

ε_{plas} strain at which maximum stress is reached (ULS)

ε_{max} assumed maximum strain (SLS)

ε_{peak} strain corresponding to (first) peak stress (SLS)

ε_{pop} strain corresponding to peak stress in Popovics curve (SLS)

ε_{β} $\varepsilon_{\beta} = (1 - \beta)\varepsilon_{u}$

ACI

The density of normal weight concrete is assumed to be 2200kg/m³.

The design strength is given in 22.2.2.4.1 by

$$f_{cd} = 0.85 \, f_{c}$$

The tensile strength is given in Equation 9-10 by

$$f_{ct} = 0.62 \sqrt{f_{c}}$$

$$f_{ct} = 7.5 \sqrt{f_{c}} \quad \text{(US units)}$$

The elastic modulus is given in 8.5.1 as

$$E = 4.7 \sqrt{f_{c}}$$

$$E = 57000 \sqrt{f_{c}} \quad \text{(US units)}$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	0.003	ε_{cu}	$(1 - 3\beta)\,\varepsilon_{cu}$		
Rectangle	0.003	ε_{cu}	ε_{β}		
Bilinear	0.003	ε_{cu}	$(1 - 2\beta)\,\varepsilon_{cu}$		
Linear				0.003	ε_{max}

FIB					
Popovics				0.003	ε_{pop}
EC2 Confined					
AISC filled tube					
Explicit	0.003	ε_{cu}		0.003	

AS

The density of normal weight concrete is taken as 2400kg/m³(3.1.3).

The design strength is given in 10.6.2.5(b) by

$$f_{cd} = \alpha_2 f_c$$

with

$$\alpha_2 = 1 - 0.003 \, f_c$$

and limits of [0.67:0.85]

The tensile strength is given in 3.1.1.3 by

$$f_{ct} = 0.6 \sqrt{f_c}$$

The elastic modulus is given (in MPa) in 3.1.2 as

$$E = \rho^{1.5} \times 0.043 \sqrt{f_{cmi}} \qquad f_{cmi} \leq 40 \, MPa$$

$$E = \rho^{1.5} \times 0.024 \sqrt{f_{cmi}} + 0.1 \qquad f_{cmi} > 40 \, MPa$$

This tabulated in Table 3.1.2.

f_c (MPa)	E (GPa)
20	24.0
25	26.7
32	30.1

40	32.8
50	34.8
65	37.4
80	39.6
100	42.2

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle					
Rectangle	0.003	0.0025	ε_β	0.003	ε_β
Bilinear					
Linear				0.003	ε_{max}
FIB					
Popovics				ε_{max}	ε_{pop}
EC2 Confined					
AISC filled tube					
Explicit	0.003	0.0025		0.003	

BS 5400

The density of normal weight concrete is given in Appendix B as 2300kg/m³.

The design strength is given in Figure 6.1 by

$$f_{cd} = 0.6\,f_c\,/\gamma$$

The tensile strength is given in 6.3.4.2 as

$$f_{ct} = 0.36 \sqrt{f_c}$$

but A.2.2 implies a value of 1MPa should be used at the position of tensile reinforcement.

The elastic modulus tabulated in 4.3.2.1 Table 3

f_c (MPa)	E (GPa)
20	25.0
25	26.0
32	28.0
40	31.0
50	34.0
60	36.0

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	0.0035	ε_{cu}	ε_{RP}		
Rectangle					
Bilinear					
Linear				0.0035	ε_{max}
FIB					
Popovics					
EC2 Confined					
AISC filled tube					

| Explicit | 0.0035 | ε_{cu} | | 0.0035 | |

$$\varepsilon_{RP} = 2.4 \times 10^{-4} \sqrt{\frac{f_c}{\gamma}}$$

BS 8110

The density of normal weight concrete is given in section 7.2 of BS 8110-2 as 2400kg/m³.

The design strength is given in Figure 3.3 by

$$f_{cd} = 0.67 \, f_c / \gamma$$

The tensile strength is given in 4.3.8.4 as

$$f_{ct} = 0.36 \sqrt{f_c}$$

but Figure 3.1 in BS 8110-2 implies a value of 1MPa should be used at the position of tensile reinforcement.

The elastic modulus is given in Equation 17

$$E = 20 + 0.2 \, f_c$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	ε_u	ε_{cu}	ε_{RP}	0.0035*	ε_{RP}
Rectangle	ε_u	ε_{cu}	ε_β		
Bilinear					
Linear				ε_u	ε_{max}
FIB				ε_u	0.0022
Popovics					
EC2 Confined					

AISC filled tube					
Explicit	ε_u	ε_{cu}		ε_u	

$$\varepsilon_u = \begin{cases} 0.0035 & f_c \leq 60 \ MPa \\ 0.0035 - 0.001 \times \dfrac{(f_c - 60)}{50} \end{cases}$$

$$\varepsilon_{RP} = 2.4 \times 10^{-4} \sqrt{\frac{f_c}{\gamma}}$$

CSA A23.3 / CSA S6

The density of normal weight concrete is assumed to be 2300 kg/m³; see 8.6.2.2 (A23.3) and 8.4.1.7 (S6).

The design strength is given in 10.1.7 by

$$f_{cd} = \max \ (0.67, 0.85 - 0.0015 \ f_c) \varphi f_c$$

The tensile strength is given in Equation 8.3 (A23.3) and 8.4.1.8.1 in (S6)

$$f_{ct} = 0.6 \sqrt{f_c} \quad \text{(for CSA A23.3)}$$

$$f_{ct} = 0.4 \sqrt{f_c} \quad \text{(for CSA S6)}$$

For normal weight concrete the modulus is given in A23.3 Equation 8.2.

$$E = 4.5 \sqrt{f_c}$$

and in CSA S6 8.4.1.7

$$E = 3.0 \sqrt{f_c} + 6.9$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	0.0035	ε_{cu}	$(1 - 3\beta) \varepsilon_u$		
Rectangle	0.0035	ε_{cu}	ε_β		

Bilinear	0.0035	ε_{cu}	$(1 - 2\beta)\,\varepsilon_u$		
Linear				0.0035	ε_{max}
FIB					
Popovics				0.0035	ε_{pop}
EC2 Confined					
AISC filled tube					
Explicit	0.0035	ε_{cu}		0.0035	

EN 1992

The density of normal weight concrete is specified in 11.3.2 as 2200 kg/m³.

The design strength is given in 3.1.6 by

$$f_{cd} = \alpha_{cc}\, f_c / \gamma$$

For the rectangular stress block this is modified to

$$f_{cd} = \alpha_{cc}\, f_c / \gamma \qquad\qquad f_c \leq 50\ MPa$$

$$f_{cd} = \alpha_{cc}\, 1.25\,(1 - f_c / 250\,)f_c / \gamma \qquad\qquad f_c > 50\ MPa$$

The tensile strength is given in Table 3.1 as

$$f_{ct} = 0.3\, f_c^{2/3} \qquad\qquad f_c \leq 50\ MPa$$

$$f_{ct} = 2.12\ \ln\left(1 + (f_c + 8)/10\,\right) \qquad\qquad f_c > 50\ MPa$$

The modulus is defined in Table 3.1

$$E = 22\left(\frac{f_c + 8}{10}\right)^{0.3}$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}

Parabola-rectangle	ε_{cu2}	ε_{c2}	ε_{c2}		
Rectangle	ε_{cu3}	ε_{c3}	ε_{β}		
Bilinear	ε_{cu3}	ε_{c3}	ε_{c3}	ε_{cu3}	ε_{c3}
Linear				ε_{cu2}	ε_{c2}
FIB				ε_{cu1}	ε_{c1}
Popovics					
EC2 Confined	$\varepsilon_{cu2,c}$	$\varepsilon_{c2,c}$	$\varepsilon_{c2,c}$		
AISC filled tube					
Explicit	ε_{cu2}	ε_{cu2} ?		ε_{cu2}	

$$\varepsilon_{c1} = 0.007 \; f_{cm}^{\,0.31} \qquad \leq 0.0028$$

$$\varepsilon_{cu1} = \begin{cases} 0.0035 & f_c \leq 50 \; MPa \\ 0.0028 + 0.027 \left(\dfrac{90 - f_c}{100} \right)^4 \end{cases}$$

$$\varepsilon_{c2} = \begin{cases} 0.002 & f_c \leq 50 \; MPa \\ 0.002 + 0.000085 \; \left(f_{ck} - 50 \right)^{0.53} \end{cases}$$

$$\varepsilon_{cu2} = \begin{cases} 0.0035 & f_c \leq 50 \; MPa \\ 0.0026 + 0.035 \left(\dfrac{90 - f_c}{100} \right)^4 \end{cases}$$

$$\varepsilon_{c3} = \begin{cases} 0.00175 & f_c \leq 50 \; MPa \\ 0.00175 + 0.00055 \; \left(\dfrac{f_{ck} - 50}{40} \right) \end{cases}$$

$$\varepsilon_{cu3} = \begin{cases} 0.0035 & f_c \leq 50 \; MPa \\ 0.0026 + 0.035 \left(\dfrac{90 - f_c}{100} \right)^4 \end{cases}$$

HK CP

The density of normal weight concrete is assumed to be 2400kg/m³.

The design strength is given in Figure 6.1 by

$$f_{cd} = 0.67 \, f_{c}/\gamma$$

The tensile strength is given in 12.3.8.4 as

$$f_{ct} = 0.36 \sqrt{f_c}$$

but 7.3.6 implies a value of 1MPa should be used at the position of tensile reinforcement.

The elastic modulus is defined in 3.1.5

$$E = 3.46 \sqrt{fc} + 3.21$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	ε_u	ε_{cu}	ε_{RP}		
Rectangle	ε_u	ε_{cu}	ε_β		
Bilinear					
Linear				ε_u	ε_u
FIB				ε_u	0.0022
Popovics					
EC2 Confined					
AISC filled tube					
Explicit	ε_u	ε_{cu}		ε_u	

$$\varepsilon_u = 0.0035 - 0.00006 \times \sqrt{f_c - 60} \qquad f_c > 60 \, MPa$$

$$E_d = 3.46 \sqrt{\frac{f_c}{\gamma}} + 3.21 \ GPa$$

$$\varepsilon_{RP} = 1.34 \ \frac{f_c/\gamma}{E_d}$$

HK SDM

The density of normal weight concrete is assumed to be 2400kg/m³.

The design strength is given in 5.3.2.1(b) of BS 5400-4 by

$$f_{cd} = 0.6 \ f_c/\gamma$$

The tensile strength is given in 6.3.4.2 as

$$f_{ct} = 0.36 \ \sqrt{f_c}$$

but from BS5400 a value of 1MPa should be used at the position of tensile reinforcement.

The elastic modulus is tabulated in Table 21

f_c (MPa)	E (GPa)
20	18.9
25	20.2
32	21.7
40	24.0
45	26.0
50	27.4
55	28.8
60	30.2

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	0.0035	ε_{cu}	ε_{RP}		

Rectangle					
Bilinear					
Linear				0.0035	ε_{max}
FIB					
Popovics					
EC2 Confined					
AISC filled tube					
Explicit	0.0035	ε_{cu}		0.0035	

$$\varepsilon_{RP} = 2.4 \times 10^{-4} \sqrt{\frac{f_c}{\gamma}}$$

IRC 112

The density of normal eight concrete is assume to be 2200kg/m³.

The design strength is given in 6.4.2.8

$$f_{cd} = 0.67 \, f_c / \gamma$$

In A2.9(2) the strength is modified for the rectangular stress block to

$$f_{cd} = 0.67 \, f_c / \gamma \qquad\qquad f_c \leq 60 \; MPa$$

$$f_{cd} = 0.67 \left(1.24 - f_c / 250 \right) f_c / \gamma \qquad\qquad f_c > 60 \; MPa$$

The tensile strength is given in by A2.2(2) by

$$f_{ct} = 0.259 \, f_c^{2/3} \qquad\qquad f_c \leq 60 \; MPa$$

$$f_{ct} = 2.27 \ln \left(1 + \left(f_c + 10\right)/12.5\right) \qquad\qquad f_c > 60 \; MPa$$

The elastic modulus is given in A2.3, equation A2-5

$$E = 22 \left(\frac{f_c + 10}{12.5}\right)^{0.3}$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	ε_{cu2}	ε_{c2}	ε_{c2}	ε_{cu2}	ε_{c2}
Rectangle	ε_{cu3}	ε_{c3}	ε_{β}		
Bilinear	ε_{cu3}	ε_{c3}	ε_{c3}	ε_{cu3}	ε_{c3}
Linear				ε_{cu2}	ε_{c2}
FIB				ε_{cu1}	ε_{c1}
Popovics					
EC2 Confined	$\varepsilon_{cu2,c}$	$\varepsilon_{c2,c}$	$\varepsilon_{c2,c}$		
AISC filled tube					
Explicit	ε_{cu2}	ε_{cu2} ?		ε_{cu2}	

$$\varepsilon_{c1} = 0.00653 \; \left(f_c + 10 \right)^{0.31} \qquad \le 0.0028$$

$$\varepsilon_{cu1} = \begin{cases} 0.0035 & f_c \le 60 \; MPa \\ 0.0028 \; + 0.027 \; \left(\dfrac{90 \; - 0.8 \; f_c}{100} \right)^4 \end{cases}$$

$$\varepsilon_{c2} = \begin{cases} 0.002 & f_c \le 60 \; MPa \\ 0.002 \; + 0.000085 \; \left(0.8 \; f_{ck} \; - 50 \right)^{0.53} \end{cases}$$

$$\varepsilon_{cu2} = \begin{cases} 0.0035 & f_c \le 60 \; MPa \\ 0.0026 \; + 0.035 \; \left(\dfrac{90 \; - 0.8 \; f_c}{100} \right)^4 \end{cases}$$

$$\varepsilon_{c3} = \begin{cases} 0.00175 & f_c \le 60 \; MPa \\ 0.00175 \; + 0.00055 \; \left(\dfrac{0.8 \; f_{ck} \; - 50}{40} \right) \end{cases}$$

$$\varepsilon_{cu\,3} = \begin{cases} 0.0035 & f_c < 50 \; MPa \\ 0.0026 \;\; + 0.035 \left(\dfrac{90 \;-\; 0.8 \, f_c}{100} \right)^4 \end{cases}$$

IRS Bridge

The density is assumed to be 2300kg/m³.

The design strength is given in 15.4.2.1(b) by

$$f_{cd} = 0.6 \, f_c / \gamma$$

The tensile strength is given in 16.4.4.3 as

$$f_{ct} = 0.37 \sqrt{f_c}$$

The elastic modulus is tabulated in 5.2.2.1

f_c (MPa)	E (GPa)
20	25.0
25	26.0
32	28.0
40	31.0
50	34.0
60	36.0

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	0.0035	ε_{cu}	ε_{RP}		
Rectangle					
Bilinear					

Linear				0.0035	ε_{max}
FIB					
Popovics					
EC2 Confined					
AISC filled tube					
Explicit	0.0035	ε_{cu}		0.0035	

$$\varepsilon_{RP} = 2.4 \times 10^{-4} \sqrt{\frac{f_c}{\gamma}}$$

IRC 456

The density is assumed to be 2200 kg/m³.

The design strength is given in Figure 21by

$$f_{cd} = 0.67\, f_c / \gamma$$

The tensile strength is inferred from 6.2.2 as

$$f_{ct} = 0.7 \sqrt{f_c}$$

The elastic modulus is defined in 6.2.3.1

$$E = 5 \sqrt{f_c}$$

The strains are defined as

	ε_{cu}	ε_{ax}	ε_{plas}	ε_{max}	ε_{peak}
Parabola-rectangle	0.0035	0.002	0.002		
Rectangle	0.0035	0.002	ε_{β}		
Bilinear					

Linear				0.0035	ε_{max}
FIB				0.0035	0.0022
Popovics					
EC2 Confined					
AISC filled tube					
Explicit	0.0035	0.002		0.0035	

Rebar material models

Symbols

f	rebar stress
f_y	rebar strength
f_u	rebar ultimate strength
ε	rebar strain
ε_p	strain at which rebar stress is maximum
ε_u	strain at which rebar fails

Rebar material models for different codes

Different material models are available for different design codes. These are summarised below:

	ACI 318	AS 3600	BS 5400	BS 8110	CSA A23.3	CSA S6	EN 1992	HK CP	HK SDM	IRC:112	IRS Bridge	IS 456
Elastic-plastic	•	•		•	•	•	•	•		•	•	•
Elastic-hardening							•			•		
BS 5400			•						•			
Pre-stress			•	•					•		•	
Progressive yield											•	•
Park	•											
Linear	•	•	•	•	•	•	•	•	•	•	•	•

No-compression	●	●	●	●	●	●	●	●	●	●	●	●
ASTM strand					●	●						
Explicit	●	●	●	●	●	●	●	●	●	●	●	●

Elastic-plastic

The initial slope is defined by the elastic modulus, E. Post-yield the stress remains constant until the failure strain, ε_u, is reached.
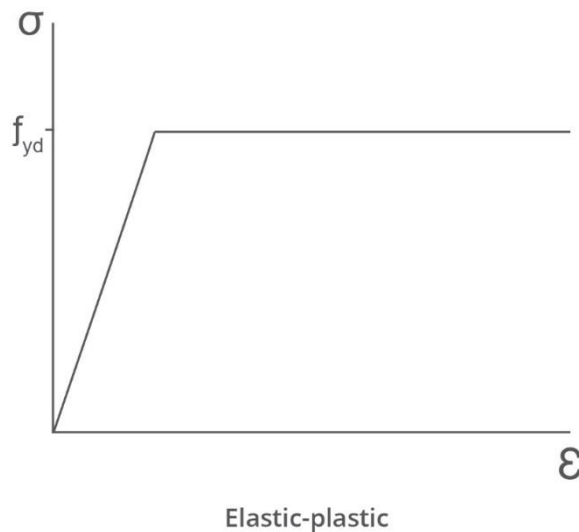
Elastic-plastic

For some codes (CAN/CSA) the initial slope is reduced to ϕE.

Elastic-hardening

The initial slope is defined by the elastic modulus, E, after yield the hardening modulus E_h governs as stress rises from $\left(\varepsilon_y, f_{yd}\right)$ to $\left(\varepsilon_u, f_u\right)$. For EN 1992 the hardening modulus is defined in terms of a hardening coefficient k and the final point is $\left(\varepsilon_{uk}, kf_{yd}\right)$ where the failure strain is reduced to ε_{ud} (typically $0.9\varepsilon_{uk}$).

The relationship between hardening modulus and hardening coefficient is:

$$E_h = \frac{(k-1)f_y}{\varepsilon_{uk} - f_y/E}$$

$$k = \frac{E_h(\varepsilon_{uk} - f_y/E)}{f_y} + 1$$

Elastic-hardening
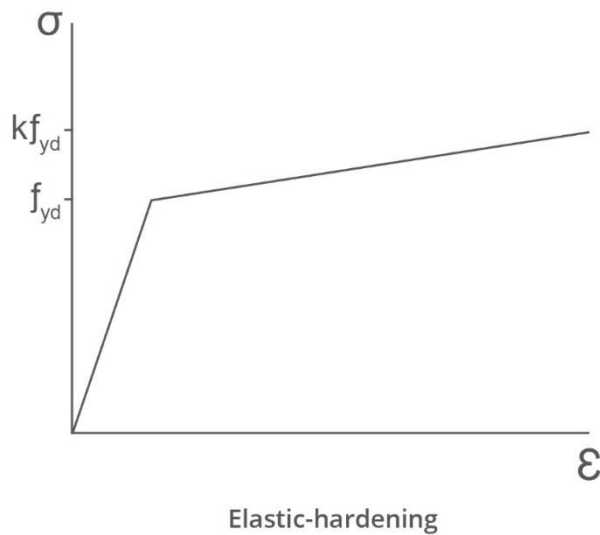
The material fails at ε_{ud} where $\varepsilon_{ud} < \varepsilon_{uk}$. This is defined in Eurocode and related codes.

BS 5400

BS 5400
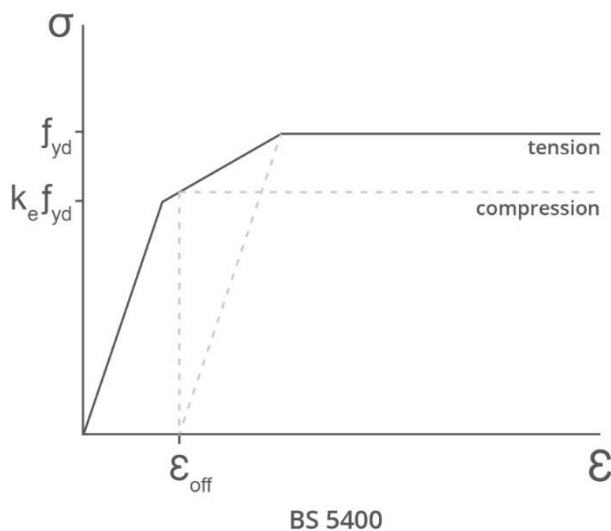
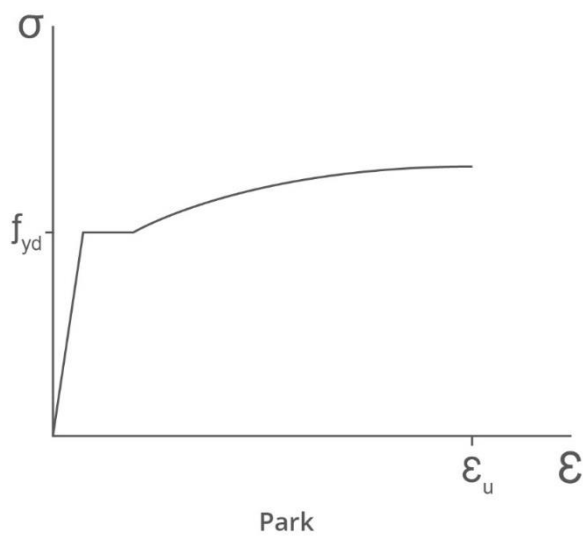In tension the initial slope is defined by the elastic modulus, E , until the stress reaches $k_e f_{yd}$. The slope then reduces until the material is fully plastic, f_{yd} , at $\varepsilon = \varepsilon_{off} + f_{yd}/E$. Post-yield the

stress remains constant until the failure strain, ε_u, is reached. For BS5400 $k_e = 0.8$ and $\varepsilon_{off} = 0.002$.

In compression the initial slope is defined by the elastic modulus, E, until the stress reaches $k_e f_{yd}$ or a strain of ε_{off} . It then follows the slope of the tension curve post-yield and when the strain reaches ε_{off} the stress remain constant until failure

Pre-stress

The initial slope is defined by the elastic modulus, E, until the stress reaches $k_e f_{yd}$. The slope then reduces until the material is fully plastic, f_{yd} , at $\varepsilon = \varepsilon_{off} + f_{yd}/E$. Post-yield the stress remains constant until the failure strain, ε_u, is reached. For BS8110 and related codes $k_e = 0.8$ and $\varepsilon_{off} = 0.005$.

Trilinear
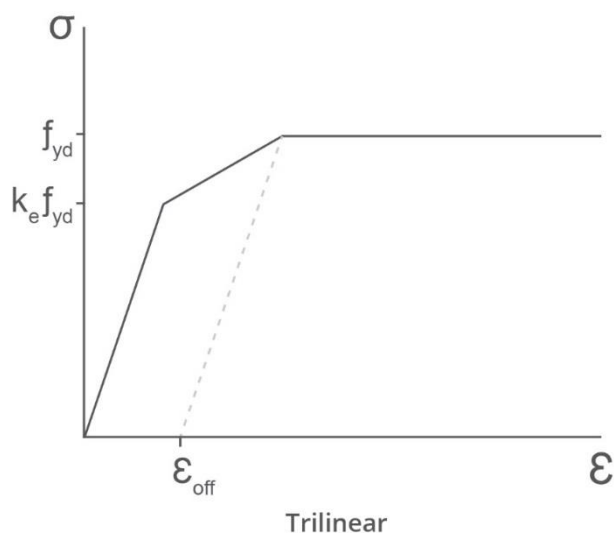
Progressive yield

The initial slope is defined by the elastic modulus, E, until the stress reaches $k_e f_{yd}$. The slope then reduces in a series of steps until the material is fully plastic, after which the stress remain constant. The points defining the progressive yield are code dependent.

Progressive yield
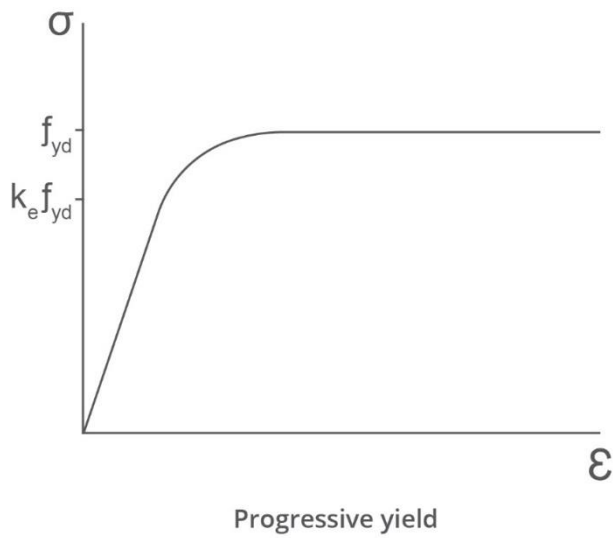
Park

The initial slope is defined by the elastic modulus, E, until the stress reaches f_{yd}. The slope is then zero for a short strain range, then rising to a peak stress before failure.

Park

$$\sigma = f_{ud} - \left(f_{ud} - f_{yd} \right) \left(\frac{\varepsilon_u - \varepsilon}{\varepsilon_u - \varepsilon_p} \right)^p$$

$$p = E \left(\frac{\varepsilon_u - \varepsilon_p}{f_{ud} - f_{yd}} \right)$$

Linear

The initial slope is defined by the elastic modulus, E , until the failure strain is reached.

No-compression

This is a linear model when in tension which has no compressive strength.

ASTM strand

The ASTM A 416 defines a stress-strain curve doe seven-wire strands. This has an initial linear relationship up to a strain of 0.008 with progressive yield till failure.
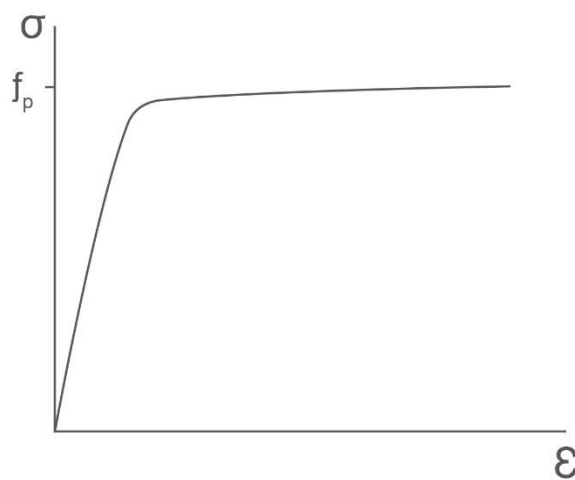
ASTM Strand

The stress strain curves are defined for specific strengths.

For Grade 250[2] (1725 MPa) the stress-strain curve is defined as

$$\sigma = 197000 \quad \varepsilon \qquad \varepsilon \leq 0.008$$

$$\sigma = 1710 \quad - \frac{0.4}{\varepsilon - 0.006} \quad \varepsilon > 0.008$$

For Grade 270 (1860 MPa) the stress-strain curve is defined as

$$\sigma = 197000 \quad \varepsilon \qquad \varepsilon \leq 0.008$$

$$\sigma = 1848 \quad - \frac{0.517}{\varepsilon - 0.003} \quad \varepsilon > 0.008$$

In the Commentary to the Canadian Bridge [3] code a similar stress-strain relationship is defined.

For Grade 1749 strand

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[2] Bridge Engineering Handbook, Ed. Wah-Fah Chen, Lian Duan, CRC Press 1999
[3] Commentary on CSA S6-14, Canadian Highway Bridge Design Code, CSA Group, 2014

$$\sigma = E_p \varepsilon \qquad\qquad \varepsilon \le 0.008$$

$$\sigma = 1749 - \frac{0.433}{\varepsilon - 0.00614} \qquad \varepsilon > 0.008$$

For Grade 1860 strand

$$\sigma = E_p \varepsilon \qquad\qquad \varepsilon \le 0.008$$

$$\sigma = 1848 - \frac{0.517}{\varepsilon - 0.0065} \qquad \varepsilon > 0.008$$

A more detailed discussion of modelling strands can be found in the paper[4] by Devalapura and Tadros

Creep

EN 1992-1-1

The creep coefficient is defined as

$$\varphi(t, t_0) = \varphi_0 \cdot \beta(t, t_0)$$

with

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

$$\varphi_{RH} = 1 + \frac{1 - RH/100}{0.1 \sqrt[3]{h_0}} \qquad \text{for } f_{cm} \le 35$$

$$\varphi_{RH} = \left[1 + \frac{1 - RH/100}{0.1 \sqrt[3]{h_0}} \cdot \alpha_1 \right] \alpha_2 \qquad \text{for } f_{cm} > 35$$

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}}$$

$$\beta(t_0) = \frac{1}{0.1 + t_0^{0.2}}$$

and

$$\beta(t, t_0) = \left[\frac{t - t_0}{\beta_H (t - t_0)} \right]^{0.3}$$

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[4] Stress-Strain Modeling of 270 ksi Low-Relaxation Prestressing Strands, Devalapura R K & Tadros M K, PCI Journal, March April 1992

And t is he age of the concrete, t_0 the age of the concrete at the time of loading and β_H a coefficient depending on the relative humidity RH as a percentage and notional member size (in mm) h_0.

$$\beta_H = 1.5\left[1 + (0.012\ RH)^{18}\right]h_0 + 250\ \le 1500 \qquad\qquad \text{for } f_{cm} \le 35$$

$$\beta_H = 1.5\left[1 + (0.012\ RH)^{18}\right]h_0 + 250\ \alpha_3 \le 1500\ \alpha_3 \qquad\qquad \text{for } f_{cm} > 35$$

The coefficients, α are

$$\alpha_1 = \left[\frac{35}{f_{cm}}\right]^{0.7} \qquad \alpha_2 = \left[\frac{35}{f_{cm}}\right]^{0.2} \qquad \alpha_3 = \left[\frac{35}{f_{cm}}\right]^{0.5}$$

and

$$h_0 = \frac{2\,A_c}{u}$$

u being the perimeter in contact with the atmosphere.

The temperature is taken into account by adjusting the times t, t_0 according to

$$t_T = \sum_{i=1}^{n} \Delta t_i\ \exp\left[-\frac{4000}{273 + T_{\Delta t_i}}\right]$$

And the cement type by modifying the times according to

$$t = t_T \left[\frac{9}{2 + t_T^{1.2}} + 1\right]^{\alpha} \ge 0.5$$

where α is

-1 for slow setting cement

0 for normal cement

1 for rapid hardening cement

AS 3600 – 2009

The creep coefficient is defined as

$$\varphi_{cc} = k_2 k_3 k_4 k_5 \varphi_{cc,b}$$

The basic creep coefficient $\varphi_{cc,b}$ is a function of concrete strength

Concrete strength, f_c' (MPa)	20	25	32	40	50	65	80	100
Basic creep coefficient $\varphi_{cc,b}$	5.2	4.2	3.4	2.8	2.4	2.0	1.7	1.5

And

$$k_2 = \frac{\alpha_2 t^{0.8}}{t^{0.8} + 0.15 t_h}$$

$$\alpha_2 = 1.0 + 1.12 \exp\left[-0.008\, t_h\right]$$

k_3 is the maturity coefficient from Figure 3.1.8.3(B). This can be tabulated as

Age of concrete at time of loading	Maturity coefficient k_3
7 days	1.76
28 days	1.1
365 days	0.9
> 365 days	0.9

k_4 is the environmental coefficient

0.70 – arid

0.65 – interior

0.6 – temperate inland

0.5 – tropical or near coast

Concrete strength factor k_5 is

$$k_5 = 1.0 \qquad \text{for } f_c' \le 50\ MPa$$

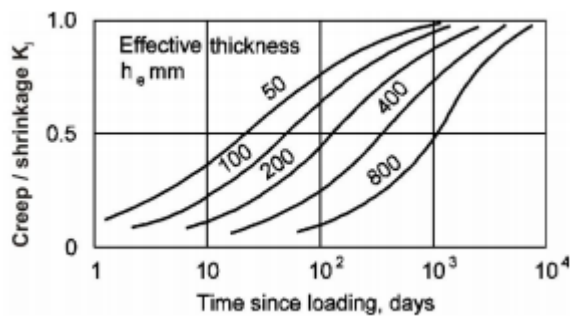$$k_5 = (2 - \alpha_3) - 0.02(1.0 - \alpha_3)f_c' \qquad \text{for } 50\ MPa < f_c' \le 100\ MPa$$
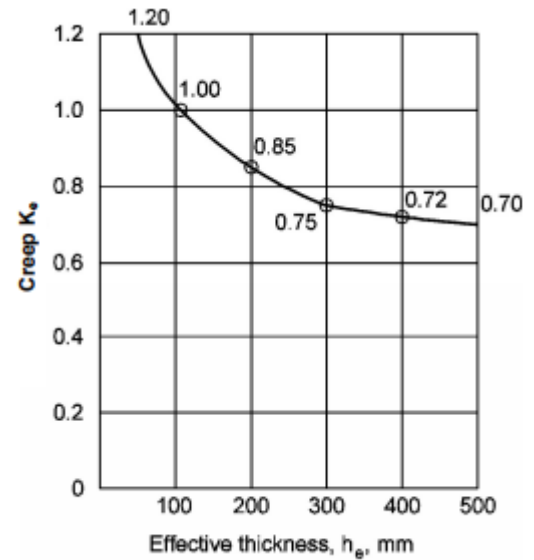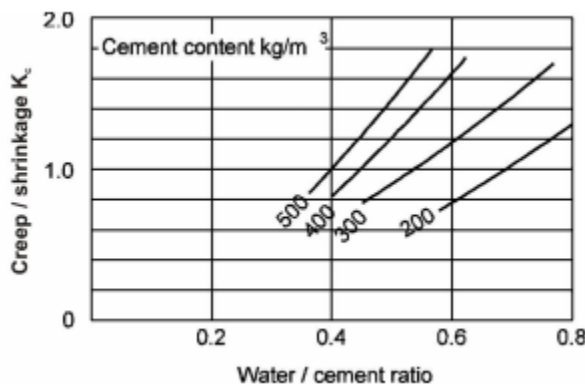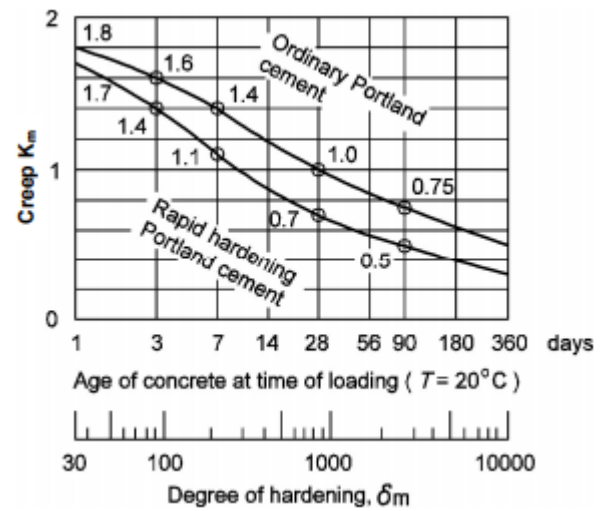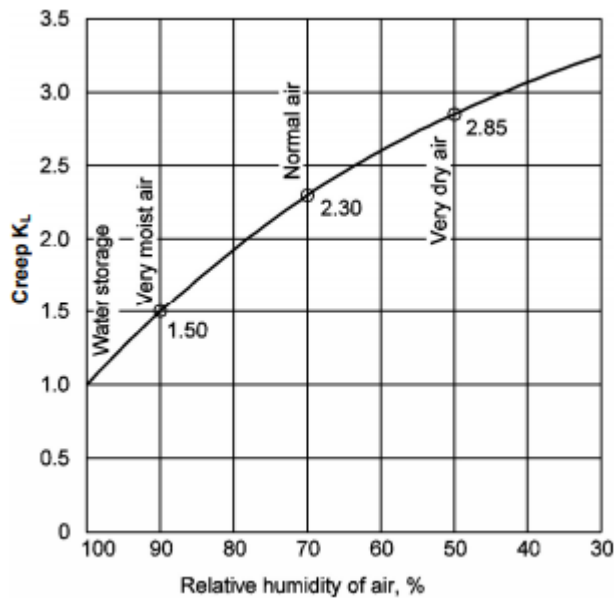
With

$$\alpha_3 = \frac{0.7}{k_4 \alpha_2}$$

Hong Kong Code of Practice

The creep coefficient is defined as

$$\varphi_c = K_L K_m K_c K_e K_j$$

Where the factors are derived from code charts.

K_m and K_j can be approximated by[5]

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[5] Allowance for creep under gradually applied loading, John Blanchard, 1998 NST 04

$$K_m(t_0) = 1 - bL + cL^2$$

with

Cement	a	b	c
OPC	1.36	0.276	0.0132
RHPC	1.09	0.333	0.0366

and

$$Kj(t - t_0) = \frac{(t - t_0)^\alpha}{\beta + (t - t_0)^\alpha}$$

with

Effective thickness (mm)	50	100	200	400	800
α	0.72	0.8	0.88	0.94	1
β	2.1	4.7	12.6	39.4	163

where t, t_0 are defined in weeks

ACI 209.2R-18

The creep coefficient is defined in Appendix A as

$$\varphi(t, t_0) = \frac{(t - t_0)^\psi}{d + (t - t_0)^\psi} \varphi_u$$

Where t is the concrete age, t_0 the age at loading and d is the average thickness and ψ are constants for member shape and size.

$$\varphi_u = 2.35 \, \gamma_c$$

$$\gamma_c = \gamma_{c,t_0} \gamma_{c,RH} \gamma_{c,d} \gamma_{c,s} \gamma_{c,\psi} \gamma_{c,\alpha}$$

with

$$\gamma_{c,t0} = 1.25 \, t_0^{-0.118} \qquad \text{for moist curing}$$

$$\gamma_{c,t0} = 1.13 \, t_0^{-0.094} \qquad \text{for steam curing}$$

$$\gamma_{c,RH} = 1.27 - 0.67 \, h \qquad \text{for relative humidity,} \, h \geq 0.4$$

$$\gamma_{c,d} = 1.14 - 0.00092 \, d \qquad \text{for} \, (t - t_0) \leq 1 \, \text{year}$$

$$\gamma_{c,d} = 1.10 - 0.00067 \, d \qquad \text{for} \, (t - t_0) > 1 \, \text{year}$$

$$\gamma_{c,s} = 0.82 + 0.00264 \ s \qquad \text{where } s \text{ is the slump in mm}$$

$$\gamma_{c,\psi} = 0.88 + 0.0024 \ \psi \qquad \text{where } \psi \text{ is the ratio of fine to total aggregate}$$

$$\gamma_{\alpha} = 0.46 + 0.09 \ \alpha \qquad \text{where } \alpha \text{ is the air content in percent}$$

IRC : 112-2011

The creep coefficient is defined as

$$\varphi(t, t_0) = \varphi_0 \cdot \beta(t, t_0)$$

with

$$\varphi_0 = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta(t_0)$$

$$\varphi_{RH} = 1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}} \qquad \text{for } f_{cm} \leq 45$$

$$\varphi_{RH} = \left[1 + \frac{1 - RH/100}{0.1\sqrt[3]{h_0}} \cdot \alpha_1\right]\alpha_2 \qquad \text{for } f_{cm} \leq 45$$

$$\beta(f_{cm}) = \frac{18.78}{\sqrt{f_{cm}}}$$

$$\beta(t_0) = \frac{1}{0.1 + t_0^{0.2}}$$

and

$$\beta(t, t_0) = \left[\frac{t - t_0}{\beta_H(t - t_0)}\right]^{0.3}$$

And t is he age of the concrete, t_0 the age of the concrete at the time of loading and β_H a coefficient depending on the relative humidity as a percentage and notional member size (in mm) h_0.

$$\beta_H = 1.5\left[1 + (0.012 \ RH)^{18}\right]h_0 + 250 \leq 1500 \qquad \text{for } f_{cm} \leq 45$$

$$\beta_H = 1.5\left[1 + (0.012 \ RH)^{18}\right]h_0 + 250 \ \alpha_3 \leq 1500 \ \alpha_3 \qquad \text{for } f_{cm} > 45 \ ^6$$

---

[6] Code specifies a value of 35MPa, but 45MPa seems more correct.

The coefficients, α are

$$\alpha_1 = \left[\frac{43.75}{f_{cm}}\right]^{0.7} \qquad \alpha_2 = \left[\frac{43.75}{f_{cm}}\right]^{0.2} \qquad \alpha_3 = \left[\frac{43.75}{f_{cm}}\right]^{0.5}$$

and

$$h_0 = \frac{2A_c}{u}$$

u being the perimeter in contact with the atmosphere.

The temperature is taken into account by adjusting the times t, t_0 according to

$$t_T = \sum_{i=1}^{n} \Delta t_i \exp\left[13.65 - \frac{4000}{273 + T_{\Delta t_i}}\right]$$

And the cement type by modifying the times according to

$$t = t_T\left[\frac{9}{2 + t_T^{1.2}} + 1\right]^{\alpha} \geq 0.5$$

where α is

-1 for slow setting cement

0 for normal cement

1 for rapid hardening cement

IS 456 : 2000

The creep coefficient is defined as a function of age at loading

Age at loading	Creep coefficient
7 days	2.2
28 days	1.6
1 year	1.1

IRS Concrete Bridge Code : 1997

The creep coefficient is defined in 5.2.4.2 as a function of age at loading

Age at loading	Creep coefficient
7 days	2.2
28 days	1.6
1 year	1.1