Frequency domain features of LS-DYNA® towards NVH and Durability analysis

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Outline

1. Introduction
   - Basics of NVH and Durability analysis
   - New Frequency Domain Features in LS-DYNA
   - New Databases in Frequency Domain

2. NVH analysis
   - Modal Analysis
   - Frequency Response Functions
   - Steady State Dynamics
   - Random Vibration
   - Acoustics and acoustic FRF

3. Durability analysis
   - Overview of Random Fatigue Feature
   - Analysis methods
   - Examples

4. Conclusion and Future Work
1. INTRODUCTION
What is NVH?

NVH stands for **Noise, Vibration** and **Harshness**.

**Noise:**
In common use, the word “Noise” means any unwanted sound. Acoustic noise can be anything from low-level but annoying to loud and harmful. It is also defined as any unpleasant or unexpected sound created by a vibrating object. The human audible sound is in 20-20000 Hz.

**Vibration:**
Vibration is defined as any objectionable repetitive motion of an object, back-and-forth or up-and-down.

**Harshness:**
Harshness refers to the qualitative assessment of noise and vibration.
More about NVH

- NVH analysis provides essential benefits towards designing vehicles for ride comfort and quietness.
- The body structure is the most significant transfer path for road-excited and engine-excited vibrations.
- The body panel vibrations are then radiated into the vehicle cavity, where they are perceived as noise.
- Apart from audible noise, which is transferred into the cavity, structural vibrations of the body are perceived as an annoyance as well.
- FEM computation studies the NVH performance before the first vehicle prototype is built. It provides even NVH diagnosis for existing vehicles.
What is Durability?

Durability is structure’s ability to survive from all kinds of loading encountered during its design life.

- For vehicle, durability is its ability to survive from severe conditions such as driving over a curb, to normal operation loading that is repeated millions of times during the life (e.g. running, accelerating, and braking);
- Durability is one of the principal concerns in the design of modern vehicles;
- Durability is more related to failure due to fatigue, other than failure due to plastic deformation, or fracture due to high stress beyond material’s strength (e.g. tensile stress limit, or yield stress limit).
Application of LS-DYNA in automotive industry

One code strategy

• In automotive, one model for crash, durability, NVH shared and maintained across analysis groups

• Manufacturing simulation results from LS-DYNA used in crash, durability, and NVH modeling.

Crashworthiness

Occupant Safety

NVH

Durability
Earlier practice of NVH and durability analysis by LS-DYNA


*They were all based on time domain simulation.*
Keywords for frequency domain analysis

*FREQUENCY_DOMAIN_FRF
*FREQUENCY_DOMAIN_SSD
*FREQUENCY_DOMAIN_RANDOM_VIBRATION_{FATIGUE}
*FREQUENCY_DOMAIN_ACOUSTIC_BEM_{PANEL_CONTRIBUTION}
*FREQUENCY_DOMAIN_ACOUSTIC_FEM
*FREQUENCY_DOMAIN_RESPONSE_SPECTRUM
Frequency domain vs. time domain

- A time-domain graph shows how a signal changes over time
- A frequency-domain graph shows the distribution of the energy (magnitude, etc.) of a signal over a range of frequencies

Frequency domain analysis
- Harmonic (steady state vibration)
- Resonance
- Linear dynamics
- Long history (fatigue testing)
- Non-deterministic load (random analysis)

Time domain analysis
- Transient analysis (penetration)
- Impact (crash simulation)
- Large deformation (fracture)
- Non-linearity (contact)
A given function or signal can be converted between the time and frequency domains with a pair of mathematical operators called a transform.

\[ H(\omega) = \int_{-\infty}^{\infty} h(t) e^{i\omega t} \, dt \]

\[ h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{-i\omega t} \, d\omega \]
Applications of the frequency domain features

- Vehicle NVH
  - Interior noise
  - Exterior radiated noise
  - Vibration

- Vehicle Durability
  - Cumulative damage ratio
  - Expected life (mileage)

- Aircraft / rocket / spacecraft vibro-acoustics

- Durability analysis of machines and electronic devices

- Acoustic design of athletic products

- Civil Engineering
  - Architectural acoustics (acoustic design of auditorium, concert hall)
  - Earthquake resistance

- Off-shore platforms, wind turbine, etc.
  - Random vibration
  - Random fatigue
New Databases in Frequency Domain

- d3ssd (lspcode=21) -> SSD
- d3spcm (lspcode=22) -> response spectrum analysis
- d3psd (lspcode=23) -> random vibration
- d3rms (lspcode=24) -> random vibration
- d3ftg (lspcode=25) -> random fatigue
- D3acs (lspcode=26) -> FEM acoustics

- frf_amplitude, frf_angle
- frf_real, frf_imag

- Press_Pa, Press_dB
- bepres
- panel_contribution_NID
- fringe_*

BEM acoustics
Keyword *DATABASE_FREQUENCY_BINARY_{OPTION}

Available options
- D3ACS
- D3FTG
- D3PSD
- D3RMS
- D3SPCM
- D3SSD

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<td>FMAX</td>
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<tr>
<td>NFREQ</td>
<td>Number of frequencies for output</td>
</tr>
<tr>
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<tr>
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<td>Load curve ID defining the frequencies for output</td>
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!![](image)

- **(Linear spacing)**
- **(Logarithmic spacing)**
- **(Biased spacing)**
2

NVH ANALYSIS
Modal Analysis

*CONTROL_IMPLICIT_GENERAL

Card 1

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Card 2 is optional

*CONTROL_IMPLICIT_EIGENVALUE

Card 1

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Intermittent eigenvalue analysis available

several options to isolate modes of interest

- lowest n modes
- n modes closest to given frequency
- all modes within given frequency range
### Mode 8

**Eigen-frequency (Hz):**
- Mode 8: 8.835
- Mode 9: 11.612
- Mode 10: 12.143
- Mode 11: 12.428
- Mode 12: 14.141
- Mode 13: 15.145
- Mode 14: 15.697
- Mode 15: 15.818
- Mode 16: 16.384
- Mode 17: 16.560
- Mode 18: 16.645
- Mode 19: 16.696
- Mode 20: 17.470
- Mode 21: 18.424
- Mode 22: 18.589

### Mode 10

**Eigen-frequency (Hz):**
- Mode 10: 12.143
- Mode 11: 12.428
- Mode 12: 14.141
- Mode 13: 15.145
- Mode 14: 15.697
- Mode 15: 15.818
- Mode 16: 16.384
- Mode 17: 16.560
- Mode 18: 16.645
- Mode 19: 16.696
- Mode 20: 17.470
- Mode 21: 18.424
- Mode 22: 18.589
**Frequency Response Functions**

**Input force** \( F(\omega) \)  

**Transfer function** \( H(\omega) \)  

**Displacement response** \( X(\omega) \)

\[
X(\omega) = H(\omega) \cdot F(\omega) \quad \quad H(\omega) = \frac{X(\omega)}{F(\omega)}
\]

- Activated by keyword *FREQUENCY_DOMAIN_FRF*
- Express structural response due to unit load as a function of frequency
- Shows the **property of the structure system**
- Is a **complex function**, with real and imaginary components. They may also be represented in terms of magnitude and phase angle
- Used in vibration analysis and modal testing
- Result files: frf_amplitude, frf_angle
- Support efficient restart
## FRF formulations

| Accelerance, Inertance | Acceleration  
<table>
<thead>
<tr>
<th></th>
<th>Force</th>
</tr>
</thead>
</table>
| Effective Mass         | Force       
|                        | Acceleration |
| Mobility               | Velocity    
|                        | Force       |
| Impedance              | Force       
|                        | Velocity    |
| Dynamic Compliance,    | Displacement|
| Admittance, Receptance | Force       |
| Dynamic Stiffness      | Force       
|                        | Displacement |
FRF configurations

- Single input / single output (SISO)
- Single input / multiple output (SIMO)
- Multiple input / single output (MISO)
- Multiple input / multiple output (MIMO)

Single Input Relationship

\[ X_p = H_{pq} F_q \]

Multiple Input Relationship

\[
\begin{bmatrix}
X_1 \\
X_2 \\
\vdots \\
X_p
\end{bmatrix}_{N_p \times 1} = 
\begin{bmatrix}
H_{11} & H_{12} & \cdots & H_{1q} \\
H_{21} & \cdots & \cdots & \cdots \\
\vdots & \cdots & \cdots & \cdots \\
H_{p1} & \cdots & \cdots & H_{pq}
\end{bmatrix}_{N_p \times N_q} \begin{bmatrix}
F_1 \\
F_2 \\
\vdots \\
F_q
\end{bmatrix}_{N_q \times 1}
\]
Application of FRF in vehicle NVH

- Locate load transfer path or energy flow for road / engine excitations
- Estimate structural properties such as dynamic stiffness, Apparent Mass (or Effective Mass)
- Locate natural frequencies, normal modes
- Basis for frequency response analysis
- Mechanical FRF and acoustic FRF
Damping options

- Constant critical damping ratio
- Mode-dependent damping ratio
- Rayleigh damping

\[ C = \alpha M + \beta K \]

- \( C \): damping matrix
- \( M \): mass
- \( K \): stiffness
- \( \alpha \): mass proportional damping coefficient
- \( \beta \): stiffness proportional damping coefficient
Example: Accelerance FRF for a plate

Harmonic point force excitation

Reference:
Point A

Point B

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<th>Mode</th>
<th>Damping ratio ($\times 0.01$)</th>
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</tr>
<tr>
<td>2</td>
<td>0.713</td>
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<tr>
<td>3</td>
<td>0.386</td>
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<td>4</td>
<td>0.328</td>
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<td>5</td>
<td>0.340</td>
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<td>6</td>
<td>0.624</td>
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<td>7</td>
<td>0.072</td>
</tr>
<tr>
<td>8</td>
<td>0.083</td>
</tr>
</tbody>
</table>
Example: nodal force/resultant force

Left end of the beam is fixed and subjected to z-directional unit acceleration

Nodal force and resultant force FRF at the left end can be obtained
Example: FRF for a trimmed BIW

Nodal force applied and displacement measured
Harmonic excitation is often encountered in engineering systems. It is commonly produced by the unbalance in rotating machinery.

The load may also come from periodical load, e.g. in fatigue test.

The excitation may also come from uneven base, e.g. the force on tires running on a zig-zag road.

May be called as
- Harmonic vibration
- Steady state vibration
- Steady state dynamics

\[ F(t) = F_0 \sin(\omega t + \phi) \]
✓ Is an extension of FRF computation
✓ Activated by keyword *FREQUENCY_DOMAIN_SSD
✓ Based on system’s eigenfrequencies and modes
✓ Results given as amplitude/phase angle, or real/imaginary pairs
✓ Binary plot file d3ssd
Example: plate

Nodal displacement response (amplitude)

Displacement response (z-direction)

Point A

Displacement response (z-direction)

Point B
Enforced motion

Relative displacement method

- base acceleration loading is applied through inertial force.
- total displacement (velocity, acceleration) is obtained by adding the relative displacement (velocity, acceleration) to the base displacement (velocity, acceleration).

\[ u = u_{relative} + u_{base} \]
\[ \dot{u} = \dot{u}_{relative} + \dot{u}_{base} \]
\[ \ddot{u} = \ddot{u}_{relative} + \ddot{u}_{base} \]

Large mass method

- a very large mass \( m_L \), which is usually \( 10^5 \text{ - } 10^7 \) times of the mass of the entire structure, is attached to the nodes under excitation
- a very large nodal force \( p_L \) is applied to the excitation dof to produce the desired enforced motion.

\[ p_L = m_L \ddot{u} \]
\[ p_L = i \omega m_L \dot{u} \]
\[ p_L = -\omega^2 m_L u \]
A rectangular plate subjected to nodal acceleration excitation

- The rectangular plate is excited by z-direction acceleration at one end.
- Steady state response at the other end is desired.

**Relative disp. method**
- The excitation points are fixed in loading dof.
- The resulted response is added with base motion to get total response.

**Large mass method**
- A large mass with $10^6$ times the original mass is added to excitation points using keyword *element_mass_node_set*.
- To produce desired acceleration, nodal force is applied to the nodes in loading dof.
- The excitation points are free in loading dof.
Example: base acceleration on auto model

Modal information

Number of nodes: ≈ 290k
Number of shells: ≈ 532k
Number of solids: ≈ 3k

Base acceleration spectrum is applied to shaker table. 500 modes up to 211 Hz are used in the simulation. The response is computed up to 100 Hz.
Why we need random vibration analysis?

- The loading on a structure is not known in a definite sense
- Many vibration environments are not related to a specific driving frequency (may have input from multiple sources)
- Provide input data for random fatigue and durability analysis

Examples:

- Fatigue
- Wind-turbine
- Air flow over a wing or past a car body
- Acoustic input from jet engine exhaust
- Earthquake ground motion
- Wheels running over a rough road
- Ocean wave loads on offshore platforms

Based on Boeing’s N-FEARA package
Correlation of multiple excitations

The multiple excitations on structure can be

- uncorrelated
- partially correlated
- fully correlated

(cross PSD function needed)

**Keyword**

*FREQUENCY_DOMAIN_RANDOM_VIBRATION*

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</table>

When SID and STYPE are both < 0, they give the IDs of correlated excitations
Benchmark Example: plate

**Thickness** = 0.05 m

**Boundary conditions**

- $X = Y = R_z = 0$ at all nodes;
- $Z = 0$ along all 4 edges;
- $R_x = 0$ along edges $x = 0$ & $x = 10$ m;
- $R_y = 0$ along edges $y = 0$ & $y = 10$ m;

**Material properties**

- $E = 200 \times 10^9$ N/m²;
- $\nu = 0.3$;
- $\rho = 8000$ kg/m³;

**Damping**

- $\zeta = 2\%$

**Reference**

Displacement-z PSD response at center

<table>
<thead>
<tr>
<th></th>
<th>Peak Frequency (Hz)</th>
<th>PSD displacement (mm²/Hz)</th>
<th>PSD stress ((N/mm²)^2)/Hz</th>
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<tr>
<td>LS-DYNA</td>
<td>2.375</td>
<td>2065.9</td>
<td>1024.7</td>
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<tr>
<td>Analytical</td>
<td>2.377</td>
<td>2063.20</td>
<td>1025.44</td>
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**Note:** Lobatto integration (3 points) was used to get stress results on shell surface.
The model is a simple pipe with 83700 elements and 105480 nodes. It is subjected to base acceleration PSD 1) in x-direction; 2) in x, y and z-directions. In ANSYS computation, solid 185 is used; in LS-DYNA, solid 18 is used.

### RMS of Von Mises stress

*(the pipe is subject to x-directional excitation only)*

<table>
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<th>Freq (hz)</th>
<th>Acceleration psd (g^2/Hz)</th>
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<td>500</td>
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<td>1000</td>
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<td>2000</td>
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Example and ANSYS results provided by Parker Hannifin Corp.
**RMS of Von Mises stress**

(the pipe is subject to x, y, z directional excitations simultaneously, uncorrelated)
The tube was fixed to the shaker tables using aluminum blocks which surrounded the tube and were tightened using screws.

Wall thickness = 3.3 mm

*Courtesy of Rafael, Israel.*
A “White Noise”, PSD of acceleration, was applied to the tube using the two shaker tables.
PSD of acceleration - CH 3 - Test results vrs. Ls-Dyna results with simple and improved fixture

- **Test**
- **Ls-Dyna with simple fixture**
- **Ls-Dyna with improved fixture**

Frequency [Hz] on the x-axis and PSD [g^2/Hz] on the y-axis.
Random vibration analysis with MPP

Nodes: 800k
Elements: 660k
Modes: 1000

Model courtesy of Predictive Engineering
Acoustics and acoustic FRF

*FREQUENCY_DOMAIN_ACOUSTIC_BEM_{OPTION}*

Available options include:

- PANEL_CONTRIBUTION
- HALF_SPACE

- **Structure loading**
  - *FEM dynamic analysis*

- **Velocities (and pressure) in time domain**
  - *FFT*

- **Velocities (and pressure) in frequency domain**
  - *BEM acoustic analysis*

- **Acoustic pressure and SPL (dB) at field points**
BEM (accurate)

- Indirect variational boundary element method
- Collocation boundary element method

They used to be time consuming

A fast solver based on domain decomposition
MPP version

Approximate (simplified) methods

- Rayleigh method
- Kirchhoff method

Assumptions and simplification in formulation
Very fast since no equation system to solve
Acoustic field generated by a vibrating plate

0.9 m by 0.6 m
Excited by uniform harmonic velocity 1 m/s at $f = 28$ Hz
No. of DOF: 336; 651; 2501; 5551; 9801
CPU time for the case of 9801 DOF (sec.)

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<td>110</td>
<td>667</td>
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Impedance condition

Example: 1-D plane wave in a box

**Size** 1m × 1m × 1m

**Boundary conditions**
Face A: unit-amplitude normal velocity
Face B: characteristic impedance

\[ \frac{p}{v_n} = \rho c \]

Other 4 Faces: rigid (normal velocity = 0)

**Parameters**
\[ \rho = 1.21 \text{ kg/m}^3, \quad c = 343 \text{ m/s}, \quad f = 5.45901 \text{ Hz (} k = 0.1) \]

**Analytical solution**
\[ p = \rho c e^{-ikx} \]

**Sound pressure (Pa) at two field points in the box**

<table>
<thead>
<tr>
<th>Field Point</th>
<th>Analytical Solution</th>
<th>BEM Solution</th>
</tr>
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<tr>
<td>(0.25, 0.5, 0.5)</td>
<td>(414.9, -10.375)</td>
<td>(414.797, -9.217)</td>
</tr>
<tr>
<td>(0.5, 0.5, 0.5)</td>
<td>(414.511, -20.743)</td>
<td>(414.533, -19.585)</td>
</tr>
</tbody>
</table>

Example: golf club

Model information

FEM part
34412 Nodes; 27616 Solid elements

BEM part
6313 Nodes; 6272 Shell elements
Acoustic Panel Contribution

\[ p(P) = \sum_{j=1}^{N} \int_{\Gamma_j} \left( G \frac{\partial p}{\partial n} - p \frac{\partial G}{\partial n} \right) d\Gamma_j \]

\[ = \sum_{j=1}^{N} p_j(P) \]

Projection in the direction of the total pressure

A simplified tunnel model

Panel 1

Panel 2

Panel 3

Panel 4

Observation point

Panel contribution at node 5401

Frequency (Hz)

Percentage (%)
Coupling with SSD

*FREQUENCY_DOMAIN_SSD

*FREQUENCY_DOMAIN_ACOUSTIC_BEM

Step 1: User uses the steady state dynamics feature to compute the vibration response of the structure, due to harmonic loading;
Step 2: The boundary velocity or acceleration obtained in step 1 is used as input for BEM acoustic computation.
Radiated noise by a car

Harmonic nodal force applied at the top

\[ f = 21 \text{ hz} \]

\[ f = 101 \text{ hz} \]
Half-space problem

Free space Green’s function

\[
G = \frac{e^{-ikr}}{4\pi r}
\]

Half space Green’s function

\[
G_H = \frac{e^{-ikr}}{4\pi r} + R_H \frac{e^{-ikr'}}{4\pi r'}
\]

\[
R_H = \begin{cases} 
1: \text{rigid reflection plane, zero velocity} \\
-1: \text{soft reflection plane, zero sound pressure (water-air interface in underwater acoustics)}
\end{cases}
\]

Helmholtz integral equation

\[
P(\omega) = -\int_S \left( i\rho \omega v_n(\omega) G_H + p(\omega) \frac{\partial G_H}{\partial n} \right) ds
\]

The reflection plane is defined by *DEFINE_PLANE.
TL (Transmission loss) is the difference in the sound power level between the incident wave entering and the transmitted wave exiting the muffler when the muffler termination is anechoic (no reflection of sound).

\[ TL = 10 \log_{10} \frac{W_i}{W_t} \]
Three-point method

Incoming wave is given as

\[ p_i = \frac{1}{2i \sin[k(x_2 - x_1)]} \left( p_1 e^{ikx_2} - p_2 e^{ikx_1} \right) \]

\[ TL = 20 \log_{10} \left( \frac{p_i}{p_o} \right) + 10 \log_{10} \left( \frac{s_i}{s_o} \right) \]

Where, \( s_i \) and \( s_o \) are the inlet and outlet tube areas, respectively.

Four-pole method

\[
\begin{bmatrix}
  p_1 \\
  v_1 
\end{bmatrix} = \begin{bmatrix}
  A & B \\
  C & D 
\end{bmatrix} \begin{bmatrix}
  p_2 \\
  -v_2 
\end{bmatrix}
\]

The four pole parameters A, B, C, D, can be obtained from

\[
A = \frac{p_1}{p_2} \quad |v_2 = 0, v_1 = 1 \\
B = \frac{p_1}{-v_2} \quad |p_2 = 0, v_1 = 1 \\
C = \frac{v_1}{p_2} \quad |v_2 = 0, v_1 = 1 \\
D = \frac{v_1}{-v_2} \quad |p_2 = 0, v_1 = 1
\]

\[
TL = 20 \log_{10} \left( \frac{1}{2} A + B \left( \frac{1}{\rho c} \right) + C \rho c + D \right) + 10 \log_{10} \left( \frac{s_i}{s_o} \right)
\]
Muffler transmission loss by different methods

![Graph showing transmission loss vs. frequency for plane wave theory, experiment, and LS-DYNA methods.]

Cutoff frequency for plane wave theory $f = 1119$ Hz

**References**


Acoustic FRF

A simplified auto body model without any inside details

Analysis steps:
1. Modal analysis
2. Steady state dynamics
3. Boundary element acoustics

All done in one run
**FREQUENCY_DOMAIN_ACOUSTIC_FEM**

1) FEM acoustics is an alternative method for simulating acoustics. It helps predict and improve sound and noise performance of various systems. The FEM simulates the entire propagation volume -- being air or water.

2) Compute acoustic pressure and SPL (sound pressure level)

3) Output binary database: d3acs (accessible by LS-PREPOST)

4) Output ASCII database: Press_Pa and Press_dB as xyplot files

5) Output frequency range dependent on mesh size

6) Very fast since
   - One unknown per node
   - The majority of the matrix is unchanged for all frequencies
   - Using a fast sparse matrix iterative solver

Hexahedron

Tetrahedron
Example: compartment

Model information
FEM: 2688 elements
BEM: 1264 elements

Excitation of the compartment (1.4×0.5×0.6) m³ by a velocity of 7mm/s
Pressure distribution

Acoustic analysis of a simplified component
Time = 10
Contours of Z-velocity
min=-1.9625e-05, at node #109338
max=-1.7835e-05, at node #109338

Fringe Levels
1.786e-05
1.784e-05
1.781e-05
1.778e-05
1.774e-05
1.772e-05
1.770e-05
1.767e-05
1.765e-05
1.763e-05

f = 10 Hz

Acoustic analysis of a simplified component
Time = 200
Contours of Z-velocity
min=-2.4584e-09, at node #109664
max=-0.06991e-06, at node #111106

Fringe Levels
4.070e-06
3.663e-06
3.256e-06
2.850e-06
2.443e-06
2.036e-06
1.629e-06
1.222e-06
8.159e-07
4.052e-07
2.453e-09

f = 200 Hz

Acoustic analysis of a simplified component
Time = 400
Contours of Z-velocity
min=-3.52468e-08, at node #110859
max=-2.03304e-05, at node #108701

Fringe Levels
2.033e-05
1.830e-05
1.627e-05
1.424e-05
1.221e-05
1.018e-05
8.153e-06
6.124e-06
4.094e-06
2.065e-06
3.526e-08

f = 400 Hz

Acoustic analysis of a simplified component
Time = 500
Contours of Z-velocity
min=-3.3454e-08, at node #109494
max=-0.04748e-05, at node #111054

Fringe Levels
4.947e-05
4.453e-05
3.959e-05
3.464e-05
2.970e-05
2.475e-05
1.981e-05
1.497e-05
9.922e-06
4.978e-06
3.349e-09

f = 500 Hz
SPL distribution

Acoustic analysis of a simplified compa
Time = 10
Contours of X-acceleration
min=-15.992, at node=999
max=16.606, at node# 109338

f = 100 Hz

Acoustic analysis of a simplified compa
Time = 200
Contours of X-acceleration
min=35.783, at node# 110864
max=193.161, at node# 111106

f = 200 Hz

Acoustic analysis of a simplified compa
Time = 400
Contours of X-acceleration
min=-61.912, at node# 110859
max=177.132, at node# 109701

f = 400 Hz

Acoustic analysis of a simplified compa
Time = 500
Contours of X-acceleration
min=61.468, at node# 109494
max=244.897, at node# 111044

f = 500 Hz
DURABILITY ANALYSIS
Overview of Random Fatigue Feature

- **Keyword** *FREQUENCY_DOMAINRANDOM_VIBRATION_FATIGUE*
- Calculate fatigue life of structures under random vibration
- Based on S-N fatigue curve
- Based on probability distribution & Miner’s Rule of Cumulative Damage Ratio

\[ R = \sum_{i} \frac{n_i}{N_i} \]

- **Schemes:**
  - Steinberg’s Three-band technique considering the number of stress cycles at the 1\(\sigma\), 2\(\sigma\), and 3\(\sigma\) levels.
  - Dirlik method based on the 4 Moments of PSD.
  - Narrow band method
  - Wirsching method
  - ...

PDF
(probability density function)
S-N fatigue curve definition

- By *define_curve
- By equation
  \[ N \cdot S^m = a \]
  \[ \log(S) = a - b \cdot \log(N) \]

\( N \): number of cycles for fatigue failure
\( S \): stress

- Fatigue life of stress below fatigue threshold

**SNLIMT** Fatigue life for stress lower than the lowest stress on S-N curve.
EQ.0: use the life at the last point on S-N curve
EQ.1: extrapolation from the last two points on S-N curve
EQ.2: infinity.

Source of information: http://www.efunda.com
Analysis methods

✓ Steinberg’s Three-band technique

considering the number of stress cycles at the $1\sigma$, $2\sigma$, and $3\sigma$ levels.

✓ Dirlik method based on the 4 Moments of PSD.

✓ Narrow band method

✓ Wirsching method

✓ Chaudhury and Dover method

✓ Tunna method

✓ Hancock method
Steinberg’s three band technique

<table>
<thead>
<tr>
<th>Standard Deviation</th>
<th>Percentage of Occurrence</th>
<th>Number of cycles for failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1σ stress</td>
<td>68.3%</td>
<td>$N_1$</td>
</tr>
<tr>
<td>2σ stress</td>
<td>27.1%</td>
<td>$N_2$</td>
</tr>
<tr>
<td>3σ stress</td>
<td>4.33%</td>
<td>$N_3$</td>
</tr>
</tbody>
</table>

\[
n_1 = E(0) \cdot T \cdot 0.683
\]
\[
n_2 = E(0) \cdot T \cdot 0.271
\]
\[
n_3 = E(0) \cdot T \cdot 0.0433
\]

\[
E[D] = \frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3}
\]

Reference
Dirlik’s method

The n-th moment of the PSD

\[ m_n = \int_0^\infty f^n G(f) \, df \]

The root mean square

\[ \text{RMS} = \sqrt{m_0} \]

Zero-crossing frequency with positive slope

\[ E[0] = \sqrt{\frac{m_2}{m_0}} \]

Peak frequency

\[ E[P] = \sqrt{\frac{m_4}{m_2}} \]

Irregularity factor

\[ \gamma = \frac{E[0]}{E[P]} = \frac{m_2}{\sqrt{m_0 m_4}} \]
\[ E(D) = \sum_i \frac{n_i}{N_i} = \sum_i \frac{E[P]T p(S)dS}{N_i} \]

\[ p(S) = \frac{D_1 e^{-\frac{Z}{Q}} + D_2 Z e^{-\frac{Z^2}{2R^2}} + D_3 Z e^{-\frac{Z^2}{2}}}{2\sqrt{m_0}} \]

**PDF function**

*(Probability density function)*

\[ x_m = \frac{m_1}{m_0} \sqrt{\frac{m_2}{m_4}} \quad R = \frac{\gamma - x_m - D_1^2}{1 - \gamma - D_1 + D_1^2} \]

\[ Q = \frac{1.25(\gamma - D_3 - D_2 R)}{D_1} \]

\[ D_1 = \frac{2(x_m - \gamma^2)}{1 + \gamma^2} \]

\[ D_2 = \frac{1 - \gamma - D_1 + D_1^2}{1 - R} \]

\[ D_3 = 1 - D_1 - D_2 \]

**Reference**

Narrow band vs broad band

Single frequency
\[ \gamma = 1 \]

White noise
\[ \gamma = 0 \]
Narrow band method

\[ E[D] = \frac{E[P]T}{a} \int S^m p(S) dS \quad \Rightarrow \quad E[D] = \frac{E[P]T}{a} S_{eqv}^m \]

Where the equivalent stress is defined as

\[ S_{eqv} = \left[ \int S^m p(S) dS \right]^{1/m} \]

For narrow band Gaussian process,

\[ p(S) = \frac{S}{4 m_0} e^{-\frac{S^2}{8m_0}} \quad E[D] = \frac{E[P]T}{a} \left( \sqrt{2 m_0} \right)^m \Gamma(1 + m/2) \]

Where, \( \Gamma(.) \) is the Gamma function.

Reference
Wirsching method

\[ E[D]_W = \zeta W \cdot E[D]_{NB} \]

where, \( \zeta W \) is the rain-flow correction factor. It is an empirical factor derived from extensive Monte Carlo simulations that include a variety of spectral density functions. It is expressed as follows

\[ \zeta W = a_W + (1 - a_W) (1 - \lambda)^{b_W} \]

where \( a_W \) and \( b_W \) are best fitting parameters expressed as

\[ a_W = 0.926 - 0.033 \ m \]
\[ b_W = 1.587 \ m - 2.323 \]

This technique was developed with reference to offshore industry, although it has been found to be applicable to a wider range of industrial problems.

Reference
For Chaudhury and Dover method, the expected damage ratio is expressed by equation

\[ E[D] = \frac{E[P]T}{a} S_{eq}^m \]

with the equivalent stress given as

\[ S_{eq} = 2\sqrt{2m_0} \left[ \frac{\lambda^{m+2}}{2\sqrt{\pi}} \Gamma\left(\frac{m+1}{2}\right) + \frac{\gamma}{2} \Gamma\left(\frac{m+2}{2}\right) + \text{erf}(\gamma) \frac{\gamma}{2} \Gamma\left(\frac{m+2}{2}\right) \right]^{1/m} \]

where

\[ \text{erf}(\gamma) = 0.3012\gamma + 0.4916\gamma^2 + 0.9181\gamma^3 - 2.354\gamma^4 \\
- 3.3307\gamma^5 + 15.6524\gamma^6 - 10.7846\gamma^7 \]

Reference

Tunna method

\[ E(D) = \sum_i \frac{n_i}{N_i} = \sum_i \frac{E[P]T \, p(S) \, dS}{N_i} \]

\[ p(S) = \frac{S}{4 \gamma m_0} e^{-\frac{S^2}{8 \gamma m_0}} \]

For \( \gamma = 1.0 \), this formula becomes the narrow band formula. Tunna’s equation was developed with specific reference to the railway industry.

Reference

For Chaudhury and Dover method, the expected damage ratio is expressed by equation

\[
E[D] = \frac{E[P]T}{a} S_{eqv}^m
\]

with the equivalent stress given as

\[
S_{eqv} = 2\sqrt{2m_0} \left[ \gamma \Gamma(1 + m/2) \right]^{1/m}
\]

Reference

Gall DS, Hancock JW. Fatigue crack growth under narrow and broad band stationary loading. Glasgow University, Marine Technology Centre; 1985.
**Examples**

**Aluminum 2014 T6**

- $\rho = 2800 \text{ kg/m}^3$
- $E = 72,400 \text{ MPa}$
- $\nu = 0.33$

**Time of exposure:** 4 hours

**Z-acceleration PSD**

PSD = 2 $g^2$/Hz

**Nodes constrained to shaker table**

**S-N fatigue curve**
RMS of Von-Mises stress (unit: GPa)

Accumulative damage ratio
(by Steinberg’s method)
Aluminum alloy 5754
\( \rho = 2700 \text{ kg/m}^3 \)
\( E = 70,000 \text{ Mpa} \)
\( \nu = 0.33 \)

Base acceleration is applied at the edge of the hole

Acceleration PSD
(exposure time: 1800 seconds)
RMS $S_x = 35.0$ MPa at critical point

<table>
<thead>
<tr>
<th>Analysis method</th>
<th>Expected life</th>
<th>Damage ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Experiment</td>
<td>7mn 25s</td>
<td>-</td>
</tr>
<tr>
<td>Steinberg</td>
<td>4mn 10s</td>
<td>7.19</td>
</tr>
<tr>
<td>Dirlik</td>
<td>5mn 25s</td>
<td>5.54</td>
</tr>
<tr>
<td>Narrow Band</td>
<td>2mn 05s</td>
<td>14.41</td>
</tr>
<tr>
<td>Wirsching</td>
<td>5mn 45s</td>
<td>5.08</td>
</tr>
<tr>
<td>Chaudhury &amp; Dover</td>
<td>6mn 03s</td>
<td>6.03</td>
</tr>
<tr>
<td>Tunna</td>
<td>4mn 06s</td>
<td>7.31</td>
</tr>
<tr>
<td>Hancock</td>
<td>22mn 18s</td>
<td>1.35</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CODE</th>
<th>RMS Sxx</th>
</tr>
</thead>
<tbody>
<tr>
<td>ANSYS</td>
<td>33.5 MPa</td>
</tr>
<tr>
<td>RADIOSS® BULK</td>
<td>35.7 MPa</td>
</tr>
<tr>
<td>LS-DYNA</td>
<td>35.0 MPa</td>
</tr>
</tbody>
</table>
Damage ratio = 7.188

Cumulative damage ratio by Steinberg’s method

Damage ratio = 5.540

Cumulative damage ratio by Dirlik method
Experiment setup

Failure at the notched point in experiment
A model from railway application

The structure is subjected to base acceleration defined by the International Standard IEC61373 to simulate the long-life test. This standard intends to highlight any weakness which may result in problem as a consequence of operation under environment where vibrations are known to occur in service on a railway vehicle.
Area 1 - damage ratio = 25.5

Area 2 - damage ratio = 3.45
CONCLUSION & FUTURE WORK
A set of frequency domain features have been implemented, towards NVH and durability analysis of vehicles

- Frequency Response Function
- Steady State Dynamics
- Random Vibration and Fatigue
- BEM & FEM Acoustics

**Future work**

- SEA method for high frequency acoustics
- Fast multi-pole BEM for acoustics
- Fatigue analysis with strains
- Feedbacks and suggestions from users

Thank you!